THE VIBRATION MODES OF A VIADUCT MADE OF REINFORCED CONCRETE BEAMS

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ABSTRACT
Dynamic modeling aims to analyze the behavior of the viaduct to external dynamic actions from road traffic or seismic movements. In the context, two case studies of rigid elastomer supports with structural symmetries that model a viaduct are presented, emphasizing the influence of the coefficients of elasticity on the shape of their own vibration modes.

1. Case study for the Viaduct of the Transylvania Highway

It is considered the simplified model in Fig. 1.1 for the viaduct modeled as a rigid and made of 20 reinforced concrete beams, individually supported on 4 identical neoprene supports. The 20 beams are grouped on four rows (Cx axis) of

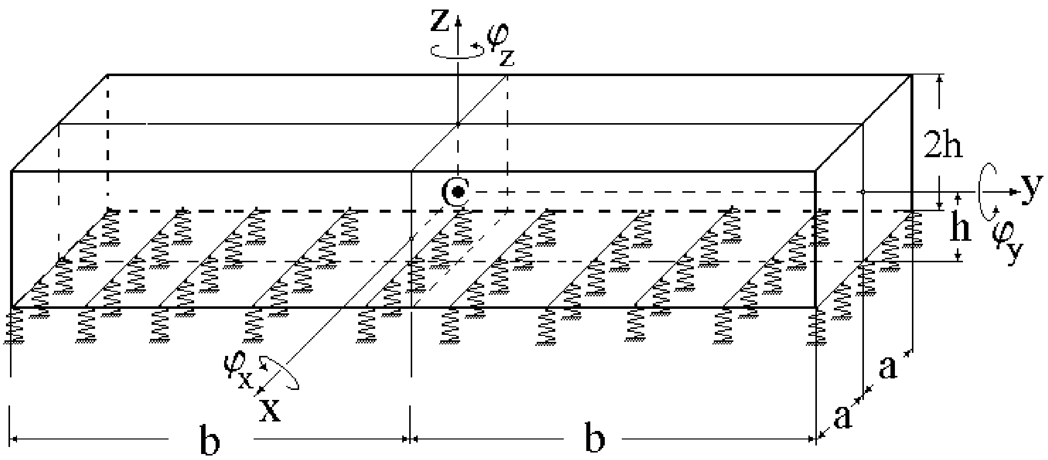

Fig 1.-1 – Modelul simplificat al viaductului cu 20 de grinzi din beton armat

5 (Cy axis), their solidarity being achieved constructively by means of over-aging. The inertial and dimensional features of the viaduct and the elasticity of the

neoprene supports are as follows: [1], [2]

■ Dimensions:

$$2b = 200m \quad 2a = 13{,}2m \quad 2h = 3m$$

■ Inertial features:

$$m = 4\times 10^{6}\,kg$$

$$J_{xy} = J_{yz} = J_{zx} = 0$$

$$J_{x} = \frac{4\times 10^{6}}{12}\left(200^{2} + 3^{2}\right) = 13{,}3363\times 10^{9}\,kgm^{2}$$

$$J_{y} = \frac{4\times 10^{6}}{12}\left(13{,}2^{2} + 3^{2}\right) = 61{,}080\times 10^{6}\,kgm^{2}$$

$$J_{z} = \frac{4\times 10^{6}}{12}\left(13{,}2^{2} + 200^{2}\right) = 13{,}3914\times 10^{9}\,kgm^{2}$$

■ Elasticity:

$$k_{ix} \equiv k_{x} = 3{,}15\times 10^{6}\,N/m \quad i = \overline{1{,}80}$$

$$k_{iy} \equiv k_{y} = 3{,}15\times 10^{6}\,N/m \quad i = \overline{1{,}80}$$

$$k_{iz} \equiv k_{z} = 650\times 10^{6}\,N/m \; i = \overline{1{,}80}$$

■ Positioning of supporting devices ([m] in the Xyz axle system) - see table 1.

Tabel 1 – Positioning of supporting devices from neopren

Support devices																			
i	**x_i**	**y_i**	**z_i**	**i**	**x_i**	**y_i**	**z_i**	**i**	**x_i**	**y_i**	**z_i**	**i**	**x_i**	**y_i**	**z_i**	**i**	**x_i**	**y_i**	**z_i**
1	-5,6	-98	-1,5	**17**	-5,6	-58	-1,5	**33**	-5,6	-18	-1,5	**49**	-5,6	22	-1,5	**65**	-5,6	62	-1,5
2	-4,3	-98	-1,5	**18**	-4,3	-58	-1,5	**34**	-4,3	-18	-1,5	**50**	-4,3	22	-1,5	**66**	-4,3	62	-1,5
3	-2,3	-98	-1,5	**19**	-2,3	-58	-1,5	**35**	-2,3	-18	-1,5	**51**	-2,3	22	-1,5	**67**	-2,3	62	-1,5
4	-1	-98	-1,5	**20**	-1	-58	-1,5	**36**	-1	-18	-1,5	**52**	-1	22	-1,5	**68**	-1	62	-1,5
5	1	-98	-1,5	**21**	1	-58	-1,5	**37**	1	-18	-1,5	**53**	1	22	-1,5	**69**	1	62	-1,5
6	2,3	-98	-1,5	**22**	2,3	-58	-1,5	**38**	2,3	-18	-1,5	**54**	2,3	22	-1,5	**70**	2,3	62	-1,5
7	4,3	-98	-1,5	**23**	4,3	-58	-1,5	**39**	4,3	-18	-1,5	**55**	4,3	22	-1,5	**71**	4,3	62	-1,5
8	5,6	-98	-1,5	**24**	5,6	-58	-1,5	**40**	5,6	-18	-1,5	**56**	5,6	22	-1,5	**72**	5,6	62	-1,5
9	-5,6	-62	-1,5	**25**	-5,6	-22	-1,5	**41**	-5,6	18	-1,5	**57**	-5,6	58	-1,5	**73**	-5,6	98	-1,5
10	-4,3	-62	-1,5	**26**	-4,3	-22	-1,5	**42**	-4,3	18	-1,5	**58**	-4,3	58	-1,5	**74**	-4,3	98	-1,5
11	-2,3	-62	-1,5	**27**	-2,3	-22	-1,5	**43**	-2,3	18	-1,5	**59**	-2,3	58	-1,5	**75**	-2,3	98	-1,5
12	-1	-62	-1,5	**28**	-1	-22	-1,5	**44**	-1	18	-1,5	**60**	-1	58	-1,5	**76**	-1	98	-1,5
13	1	-62	-1,5	**29**	1	-22	-1,5	**45**	1	18	-1,5	**61**	1	58	-1,5	**77**	1	98	-1,5
14	2,3	-62	-1,5	**30**	2,3	-22	-1,5	**46**	2,3	18	-1,5	**62**	2,3	58	-1,5	**78**	2,3	98	-1,5
15	4,3	-62	-1,5	**31**	4,3	-22	-1,5	**47**	4,3	18	-1,5	**63**	4,3	58	-1,5	**79**	4,3	98	-1,5

16	5,6	-62	-1,5	**32**	5,6	-22	-1,5	**48**	5,6	18	-1,5	**64**	5,6	58	-1,5	**80**	5,6	98	-1,5

Pulses and frequencies of non-coupled vibrations (after the six "directions") are given in Table 2.

Table 2 - Pulses and frequencies of non-coupled vibrations (after the six "directions")

Direction	X	Y	Z	φ_X	φ_Y	φ_Z
p [rad/s]	7,94	7,94	114,02	117,22	109,35	8,16
f [Hz]	1,26	1,26	18,15	18,66	17,40	1,30

The numerical expressions of the differential equation systems written in shifts of the free vibrations for the decoupled subsystems and the corresponding dynamic matrices are as follows:

- for the subsystem (X,φ_Y)

- differential motion equations $\begin{cases} 4\ddot{X} + 252X - 378\varphi_Y = 0 \; [MN] \\ 61,08\ddot{\varphi}_Y - 378X + 730387\varphi_Y = 0 \; [MNm] \end{cases}$

,

with canonical form

$$\begin{cases} \ddot{X} + 63X - 94,5\varphi_Y = 0 \; [m/s^2] \\ \ddot{\varphi}_Y - 6,1886X + 11957,875\varphi_Y = 0 \; [rad/s^2] \end{cases}$$

- the dynamic matrix $\underline{D} = \begin{bmatrix} 63 & -94,5 \\ -6,1886 & 11957,875 \end{bmatrix}$ cu UM

$$\begin{bmatrix} s^{-2} & ms^{-2} \\ m^{-1}s^{-2} & s^{-2} \end{bmatrix}$$

- for the subsystem (Y,φ_X)

- differential motion equations $\begin{cases} 4\ddot{Y} + 252Y + 378\varphi_x = 0 \; [MN] \\ 13336,3\ddot{\varphi}_x + 378Y + 183242000\varphi_x = 0 \; [MNm] \end{cases}$,

with canonical form

$$\begin{cases} \ddot{Y} + 63Y + 94,5\varphi_X = 0 \; [m/s^2] \\ \ddot{\varphi}_X + 0,028344Y + 13740,093\varphi_X = 0 \; [rad/s^2] \end{cases}$$

- the dynamic matrix $\underline{D} = \begin{bmatrix} 63 & 94{,}5 \\ 0{,}028344 & 13740{,}093 \end{bmatrix}$ cu UM $\begin{bmatrix} s^{-2} & ms^{-2} \\ m^{-1}s^{-2} & s^{-2} \end{bmatrix}$

● the jump-off movement

- the equation of motion $4\ddot{Z} + 52000Z = 0 \; [MN]$
- the canonical form of the motion equation $\ddot{Z} + 13000Z = 0 \; \left[m/s^2\right]$

● the movement off the turn

- the equation of motion $13391{,}4\ddot{\varphi}_Z + 891585\varphi_Z = 0 \; [MNm]$
- the canonical form of the motion equation $\ddot{\varphi}_Z + 66{,}5789\varphi_Z = 0 \; \left[rad/s^2\right]$

In Table 3 the values determined with the calculation relations for the pulses and frequencies of the decoupled subsystems are passed, as well as the values of the distribution coefficients $\mu_i \;\; i = \overline{1,4}$.

Table 3 - Pulses and frequencies of subsystems with decoupled movements

Subsystem	Pulsation	Frequency	Distribution coefficients
(X, φ_Y)	$p_1 = 7{,}94 rad/s$	$f_1 = 1{,}26 Hz$	$\mu_1 = -0{,}0005 rad/m$
	$p_2 = 109{,}35 rad/s$	$f_2 = 17{,}40 Hz$	$\mu_2 = -125{,}867 rad/m$
(Y, φ_X)	$p_3 = 7{,}94 rad/s$	$f_3 = 1{,}26 Hz$	$\mu_3 = 0{,}0005 rad/m$
	$p_4 = 117{,}22 rad/s$	$f_4 = 18{,}22 Hz$	$\mu_4 = 144{,}736 rad/m$
(Z)	$p_5 = p_Z = 114{,}02 rad/s$	$f_5 = f_Z = 18{,}15 Hz$	-
(φ_Z)	$p_6 = p_{\varphi_Z} = 8{,}16 rad/s$	$f_6 = f_{\varphi_Z} = 1{,}30 Hz$	-

2. Case study for the viaduct section made of 4 reinforced concrete beams (between two piles)

The simplified model of Fig. 1-2 for a section of the viaduct located between two piles of it (in total, the viaduct consists of five such sections, each section being made of four identical beams arranged longitudinally and solidarised by means of an over-crushing).

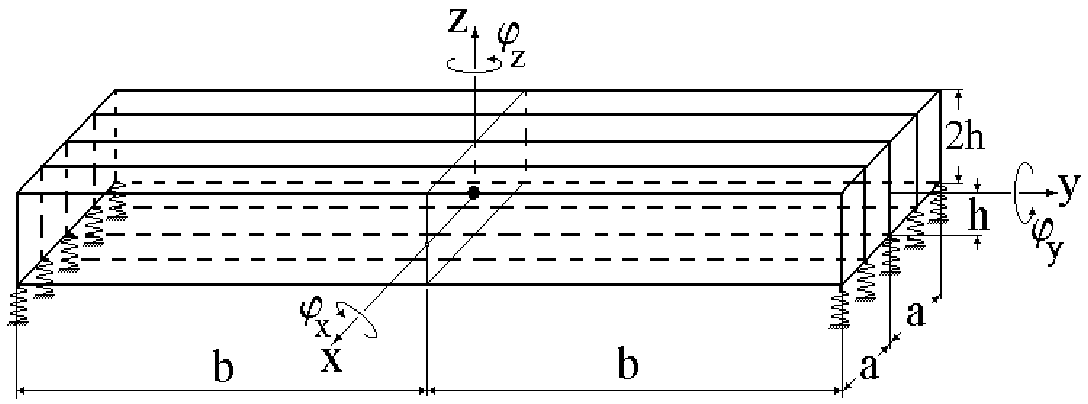

Fig. 1-2 – The simplified model of the viaduct (4 reinforced concrete beams)

each beam is supported by four identical neoprene support devices, the positions of these supports being given by the coordinates in the Cxyz system shown in Table 4.

Table 4 - Positioning of Neoprene Support devices

Support devices															
i	**x_i**	**y_i**	**z_i**	**i**	**x_i**	**y_i**	**z_i**	**i**	**x_i**	**y_i**	**z_i**	**i**	**x_i**	**y_i**	**z_i**
1	-5,6	-18	-1,5	**5**	1	-18	-1,5	**9**	-5,6	18	-1,5	**13**	1	18	-1,5
2	-4,3	-18	-1,5	**6**	2,3	-18	-1,5	**10**	-4,3	18	-1,5	**14**	2,3	18	-1,5
3	-2,3	-18	-1,5	**7**	4,3	-18	-1,5	**11**	-2,3	18	-1,5	**15**	4,3	18	-1,5
4	-1	-18	-1,5	**8**	5,6	-18	-1,5	**12**	-1	18	-1,5	**16**	5,6	18	-1,5

The numerical expressions of the differential equation systems written in free vibration displacements for the decoupled subsystems and the corresponding dynamic matrices are as follows: [1], [2]

- for the subsystem $\left(X,\varphi_y\right)$

\- differential motion equations

$$\begin{cases} 0{,}8\ddot{X}+50{,}4X-75{,}6\varphi_y=0 & [MN] \\ 12{,}216\ddot{\varphi}_y-75{,}6X+146077\varphi_y=0 & [MNm] \end{cases},$$

with canonical form

$$\begin{cases} \ddot{X}+63X-94{,}5\varphi_y=0 & \left[m/s^2\right] \\ \ddot{\varphi}_y-6{,}1886X+11957{,}842\varphi_y=0 & \left[rad/s^2\right] \end{cases}$$

\- the dynamic matrix $\underline{D}=\begin{bmatrix} 63 & -94{,}5 \\ -6{,}1886 & 11957{,}842 \end{bmatrix}$ cu UM $\begin{bmatrix} s^{-2} & ms^{-2} \\ m^{-1}s^{-2} & s^{-2} \end{bmatrix}$

- for the subsystem (Y,φ_X) - differential motion equations $\begin{cases} 0{,}8\ddot{Y}+50{,}4Y+75{,}6\varphi_X=0 & [MN] \\ 107{,}266\ddot{\varphi}_X+75{,}6Y+3369710\varphi_X=0 & [MNm] \end{cases}$, with canonical form $\begin{cases} \ddot{Y}+63Y+94{,}5\varphi_X=0 & \left[m/s^2\right] \\ \ddot{\varphi}_X+0{,}70479Y+31414{,}521\varphi_X=0 & \left[rad/s^2\right] \end{cases}$

- the dynamic matrix $\underline{D}=\begin{bmatrix} 63 & 94{,}5 \\ 0{,}70479 & 31414{,}52 \end{bmatrix}$ cu UM $\begin{bmatrix} s^{-2} & ms^{-2} \\ m^{-1}s^{-2} & s^{-2} \end{bmatrix}$

● the jump-off movement

- the equation of motion $0{,}8\ddot{Z}+10400Z=0 \ [MN]$

- the canonical form of the motion equation $\ddot{Z}+13000Z=0 \ \left[m/s^2\right]$

● the movement off the turn

- the equation of motion
$118{,}282\ddot{\varphi}_Z+17036{,}964\varphi_Z=0 \ [MNm]$

- the canonical form of the motion equation
$\ddot{\varphi}_Z+144{,}0368\varphi_Z=0 \ \left[rad/s^2\right]$

In Table 5 the values determined with the relations for the pulses and the own frequencies of the decoupled subsystems, as well as the values of the distribution coefficients $\mu_i \quad i=\overline{1,4}$.

Table 5 - Pulses and frequencies of subsystems with decoupled movements

Subsystem	Pulsation	Frequency	Distribution coefficients
(X,φ_Y)	$p_1=7{,}94rad/s$	$f_1=1{,}26Hz$	$\mu_1=-0{,}0005rad/m$
	$p_2=109{,}35rad/s$	$f_2=17{,}40Hz$	$\mu_2=-125{,}867rad/m$
(Y,φ_X)	$p_3=7{,}94rad/s$	$f_3=1{,}26Hz$	$\mu_3=0{,}0005rad/m$
	$p_4=177{,}24rad/s$	$f_4=28{,}21Hz$	$\mu_4=331{,}757rad/m$
(Z)	$p_5=p_Z=114{,}02rad/s$	$f_5=f_Z=18{,}15Hz$	-
(φ_Z)	$p_6=p_{\varphi_z}=12{,}00rad/s$	$f_6=f_{\varphi_z}=1{,}91Hz$	-

3. CONCLUSIONS

- The modeling of a rigid solid with elastic or visco-elastic bonds with different types of symmetries leads to the obtaining of systems of differential movement equations decoupled into subsystems with less coupling coefficients and thus easier to analytically studied; In this way, the influences of the dimensional, inertial, elastic (possibly damping) factors can be emphasized on the forms of the vibration modes;

- If the rigid solid can be modeled with the symmetries so that its movements relate to a central and main axis system, then its movements after the six "directions" (X, Y, Z, φ_X, φ_Y, φ_Z) are coupled only by the non-legacy coefficients of the matrix rigidity (possibly also through depreciation if significant);
- For the case study of the two rigid considered, one can find:

 - "grouping" three of their own frequencies in the range 1 ÷ 2,5 Hz, the other three frequencies being much higher than the first three and grouped in the range 17 ÷ 18,5 Hz, for the viaduct built from 20 solidarised beams;;

 - "grouping" three of their own frequencies in the range 1 ÷ 2 Hz, the other three frequencies being much higher than the first three and grouping two of them in the range 17 ÷ 18.5 Hz, the third being much higher in case of viaduct between two piles (built of 4 solidarized beams);

- this large difference between the individual frequencies can be explained by the very large difference between the elasticity of the vertical bearing elements (compressive effort) and the horizontal elasticity (shear stress) - the ratio of the elastic constants is about $\mathbf{1:206}$;
- Obtaining very high or very low values of the distribution coefficients leads to the conclusion that within the subsystems (X,φ_Y) the movements are actually very weakly coupled. In fact, it can be considered that the movements of these subsystems are quasi-decoupled (this decoupling can also be observed from the relatively small values of the coupling coefficients α_1, α_2, β_1 și β_2).

Bibliography

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