# Analysis of Active Two-Port Circuits with Variable Loads on the Basis of Projective Geometry

Alexandr PENIN
*Institute of the Electronic Engineering and Nanotechnologies "D. Ghitu" of the Academy of Sciences of Moldova*
aapenin@mail.ru

***Abstract*** **— For two-port circuit of direct current the problem of recalculation of the changeable load currents is considered. The approach on the basis of projective geometry is used for interpretation of changes or "kinematics" of the circuit regimes.**

**Changes of regime parameters are introduced otherwise, through the cross ratio of four points with use of projective coordinates.**

**Easy-to-use formulas of the recalculation of the currents, which possess the group properties at change of conductivity of the loads, are obtained. It allows expressing the final values of the currents through the intermediate changes of the currents and conductivities.**

**Disadvantages of the traditional approach, which uses the changes of resistance in the form of increments, are shown.**

**The given approach is applicable to the analysis of «flowed » form processes of the various physical natures.**

***Index Terms*** **— active two-port, changeable loads, projective coordinates, recalculation of currents**

## I. INTRODUCTION

In the theory of electric circuits, an attention is given to the circuits with variable parameters of elements. In practice, it can be the power supply systems, for example, of the direct current, are containing the power supply of finite capacity and a quantity (let it be two ) of resistive loads with variable resistances. Therefore, a mutual influence of loads on the value of their currents takes place.

One of problems of the analysis of circuits with changeable parameters of elements is obtainment of formulas of the recalculation of load currents.

Traditional approach, based on use of change of load resistances in the form of increments, is known. Recalculation of currents leads to the solution of system of the algebraic equations of a corresponding order.

Also, at a number or group of changes of these resistances, these increments should be counted concerning an initial circuit and the solution of the equations system is repeated. Therefore, such nonfulfilment of group properties (when the final result should be obtained through intermediate results) complicates recalculation and limits possibilities of this approach.

In a number of articles of the author, the approach is developed for interpretation of changes or "kinematics" of the circuit regimes on the basis of projective geometry [1- 4]. The changes of regime parameters are introduced otherwise. Therefore, as if obvious changes in a form of increments are formal and do not reflect of substantial aspect of the mutual influences: resistances → currents.

The offered approach allows obtaining the convenient formulas of recalculation

## II. PROJECTIVE COORDINATES OF AN ACTIVE TWO-PORT NETWORK

Let us give the necessary knowledge about the projective geometry for interpretation of changes of regimes of electric circuits. Consider an active two-port network with changed conductivities of loads $Y_{L1}, Y_{L2}$ in Fig.1.

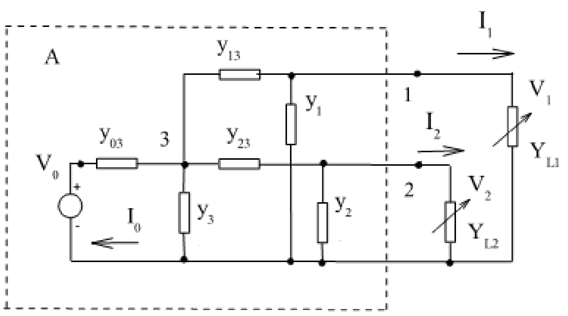


Fig.1. An active two-port with the changeable loads

Taking into account the specified directions of the currents, network is described by the following system of the Y - parameters equations:

$$,\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -Y_{11} & Y_{12} \\ Y_{12} & -Y_{22} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + \begin{pmatrix} I_1^{SC} \\ I_2^{SC} \end{pmatrix}, \qquad (1)$$

where $I_1^{SC}$, $I_2^{SC}$ are the short circuit $SC$ currents.

Taking into account the voltages $V_1 = I_1 / Y_{L1}$, $V_2 = I_2 / Y_{L2}$, two bunches of load straight lines with parameters $Y_{L1}, Y_{L2}$ are obtained in Fig.2.

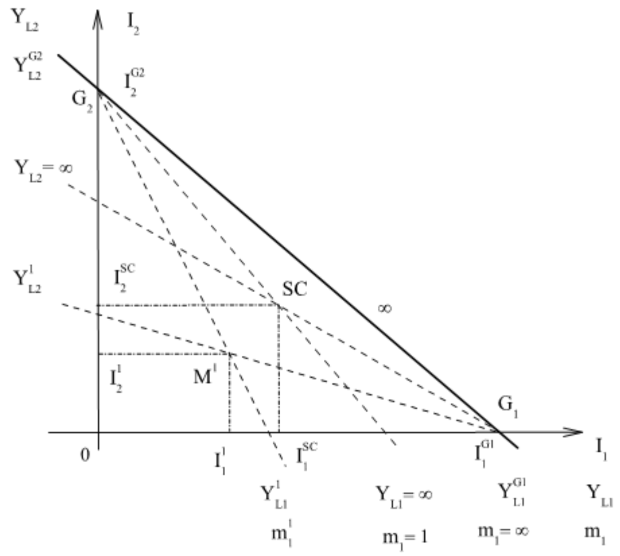


Fig.2. Two bunches of load straight lines with the parameters $Y_{L1}, Y_{L2}$

The bunch center, a point $G_2$, corresponds to the bunch with the parameter $Y_{L1}$. The bunch center corresponds to such regime of the load $Y_{L1}$ which does not depend on its values. It is carried out for the $I_1 = 0, V_1 = 0$ at the expense of a choice of the regime parameters of the second load $Y_{L2}$ :

$$I_2^{G2} = V_0\, y_{03}\left(1 + \frac{y_2}{y_{23}}\right), \; Y_{L2}^{G2} = -(y_2 + y_{23}) \qquad (2)$$

The parameters of the center $G_1$ of the bunch $Y_{L2}$ are expressed similarly.

Another form of characteristic regime is the short circuit regime of both loads ($Y_{L1} = \infty, Y_{L2} = \infty$) that is presented by the point $SC$ in Fig.2. The open circuit regime of both loads, also, is characteristic and corresponds to the origin of coordinates, the point 0.

Let the initial or running regime corresponds to the point $M^1$ which is set by the values of conductivities $Y_{L1}^1$, $Y_{L2}^1$ and currents $I_1^1$, $I_2^1$ of the loads. Also, this point is defined by the projective non-uniform $m_1^1$, $m_2^1$ and homogeneous $\xi_1^1, \xi_2^1, \xi_3^1$ coordinates which are set by a triangle of reference $G_1\ 0\ G_2$ and a unit point $SC$.

The point $0$ is the origin of coordinates and the straight line $G_1\ G_2$ is the line of infinity $\infty$.

The non-uniform projective coordinate $m_1^1$ is set by a cross ratio of four points, three of these correspond to the points of the characteristic regimes, and the fourth corresponds to the point of the running regime:

$$m_1^1 = (0\, Y_{L1}^1\, \infty\, Y_{L1}^{G1}) = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} \div \frac{\infty - 0}{\infty - Y_{L1}^{G1}}$$

$$m_1^1 = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} \qquad (3)$$

There, the points $Y_{L1} = 0$, $Y_{L1} = Y_{L1}^{G1}$ correspond to extreme or base values. The point $Y_{L1} = \infty$ is the unit point. Also, the values of $m_1$ are shown in Fig.2. For the point $Y_{L1}^1 = Y_{L1}^{G1}$, projective coordinate $m_1 = \infty$ defines the sense of line of infinity $G_1\ G_2$. The cross ratio for the projective coordinate $m_2^1$ is expressed similarly.

The homogeneous projective coordinates $\xi_1$, $\xi_2$, $\xi_3$ set the non-uniform coordinates as follows

$$m_1 = \frac{\xi_1}{\xi_3} = \frac{\rho\xi_1}{\rho\xi_3}, \; m_2 = \frac{\xi_2}{\xi_3} = \frac{\rho\xi_2}{\rho\xi_3}, \qquad (4)$$

where $\rho$ is a coefficient of proportionality.

The homogeneous coordinates are defined by the ratio of the distances of points $M^1$, $SC$ to the sides of the triangle:

$$\rho\,\xi_1^1 = \frac{\delta_1^1}{\delta_1^{SC}} = \frac{I_1^1}{I_1^{SC}}, \quad \rho\,\xi_2^1 = \frac{I_2^1}{I_2^{SC}}, \quad \rho\,\xi_3^1 = \frac{\delta_3^1}{\delta_3^{SC}}$$

For finding the distances $\delta_3^1, \delta_3^{SC}$ to the straight line $G_1\ G_2$, the equation of this straight line is used. Then

$$\left(\frac{I_1^1}{I_1^{G1}} + \frac{I_2^1}{I_2^{G2}} - 1\right) = \mu_3 \delta_3^1, \left(\frac{I_1^{SC}}{I_1^{G1}} + \frac{I_2^{SC}}{I_2^{G2}} - 1\right) = \mu_3 \delta_3^{SC}$$

where $\mu_3$ is a normalizing factor.

The homogeneous coordinates have a matrix form:

$$\rho[\xi] = [C] \cdot [I], \qquad (5)$$

Where martix

$$[C] = \begin{pmatrix} \frac{1}{I_1^{SC}} & 0 & 0 \\ 0 & \frac{1}{I_2^{SC}} & 0 \\ \frac{1}{I_1^{G1}\mu_3\delta_3^{SC}} & \frac{1}{I_2^{G2}\mu_3\delta_3^{SC}} & -\frac{1}{\mu_3\delta_3^{SC}} \end{pmatrix}$$

and one-dimensional vectors

$$[I]=\begin{pmatrix} I_1 \\ I_2 \\ 1 \end{pmatrix},[\xi\ ]=\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}$$

The inverse transformation is

$$\rho[I]=[C]^{-1}\cdot[\xi] \qquad (6)$$

Where matrix

$$[C]^{-1}=\begin{pmatrix} I_1^{SC} & 0 & 0 \\ 0 & I_2^{SC} & 0 \\ \dfrac{I_1^{SC}}{I_1^{G1}} & \dfrac{I_2^{SC}}{I_2^{G2}} & -\mu_3\delta_3^{SC} \end{pmatrix}.$$

From here, we pass to the non-uniform Cartesian coordinates or to the currents:

$$I_1=\frac{\rho I_1}{\rho 1}=\frac{I_1^{SC}\cdot m_1}{\dfrac{I_1^{SC}}{I_1^{G1}}\cdot m_1+\dfrac{I_2^{SC}}{I_2^{G2}}\cdot m_2-\mu_3\delta_3^{SC}},$$

$$I_2=\frac{I_2^{SC}\cdot m_2}{\dfrac{I_1^{SC}}{I_1^{G1}}\cdot m_1+\dfrac{I_2^{SC}}{I_2^{G2}}\cdot m_2-\mu_3\delta_3^{SC}} \qquad (7)$$

## III. Recalculation of load currents with use of projective coordinates

Let a subsequent regime corresponds to a point $M^2$ in Fig.3 with the parameters of loads $Y_{L1}^2, Y_{L2}^2, I_1^2, I_2^2$.

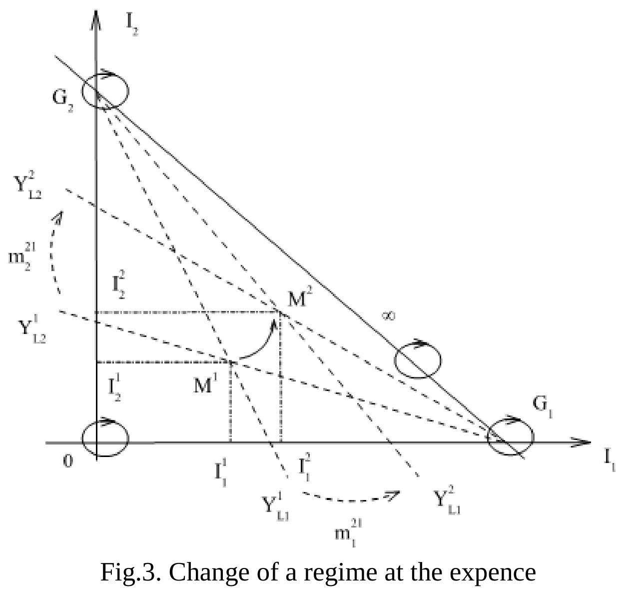


Fig.3. Change of a regime at the expence of change of the loads

The non-uniform coordinates are defined similarly to (3). Therefore, the change of the regime $m_1^{21}$ is naturally expressed through the cross ratio:

$$m_1^{21}=(0\ Y_{L1}^2\ Y_{L1}^1\ Y_{LH1}^{G1})=m_1^2:m_1^1,\ m_2^{21}=m_2^2:m_2^1$$

We also define the homogeneous coordinates of a point $M^2$.

Using the transformations (7), we receive the current

$$I_1^2=\frac{I_1^{SC}\cdot m_1^2}{\dfrac{I_1^{SC}}{I_1^{G1}}\cdot m_1^2+\dfrac{I_2^{SC}}{I_2^{G2}}\cdot m_2^2-\mu_3\delta_3^{SC}}.$$

We present the non-uniform coordinates, $m_1^2$ and $m_2^2$ in a form:

$$m_1^2=m_1^{21}\frac{\xi_1^1}{\xi_3^1},\ m_2^2=m_2^{21}\frac{\xi_2^1}{\xi_3^1}.$$

Therefore, the current

$$I_1^2=\frac{(I_1^{SC}\cdot m_1^{21})\cdot\xi_1^1/\xi_3^1}{\left(\dfrac{I_1^{SC}}{I_1^{G1}}\cdot m_1^{21}\right)\cdot\dfrac{\xi_1^1}{\xi_3^1}+\left(\dfrac{I_2^{SC}}{I_2^{G2}}\cdot m_2^{21}\right)\dfrac{\xi_2^1}{\xi_3^1}-\mu_3\delta_3^{SC}}.$$

Then, taking into account the expression (6), we receive the transformation:

$$\rho[I^2]=[C^{21}]^{-1}\cdot[\xi^1]$$

Where matrix

$$[C^{21}]^{-1}=\begin{pmatrix} I_1^{SC}\cdot m_1^{21} & 0 & 0 \\ 0 & I_2^{SC}\cdot m_2^{21} & 0 \\ \dfrac{I_1^{SC}}{I_1^{G1}}\cdot m_1^{21} & \dfrac{I_2^{SC}}{I_2^{G2}}\cdot m_2^{21} & -\mu_3\delta_3^{SC} \end{pmatrix}$$

Using (5), we receive the resultant transformation as a product of the two matrixes:

$$\rho[I^2]=[C^{21}]^{-1}\cdot[C]\cdot[I^1]=[J^{21}]\cdot[I^1] \qquad (8)$$

Where matrix

$$[J^{21}]=\begin{pmatrix} m_1^{21} & 0 & 0 \\ 0 & m_2^{21} & 0 \\ \dfrac{1}{I_1^{G1}}(m_1^{21}-1) & \dfrac{1}{I_2^{G2}}(m_2^{21}-1) & 1 \end{pmatrix}$$

From here, we pass to the required currents:

$$I_1^2 = \frac{\rho I_1^2}{\rho 1} = \frac{I_1^1 \cdot m_1^{21}}{\frac{I_1^1}{I_1^{G1}} \cdot (m_1^{21} - 1) + \frac{I_2^1}{I_2^{G2}} \cdot (m_2^{21} - 1) + 1},$$

$$I_2^2 = \frac{I_2^1 \cdot m_2^{21}}{\frac{I_1^1}{I_1^{G1}} \cdot (m_1^{21} - 1) + \frac{I_2^1}{I_2^{G2}} \cdot (m_2^{21} - 1) + 1}.$$

The obtained relationships carry out the recalculation of currents at respective change of load conductivities. These relations are the projective transformations and possess group properties.

Let a regime once again change, i.e. we receive the values of the changes $m_1^{32}, m_2^{32}$.

Then the final value of the regime is expressed as:

$$m_1^3 = m_1^{32} \cdot m_1^2 = m_1^{32} \cdot m_1^{21} \cdot m_1^1 = m_1^{31} \cdot m_1^1,$$

$$m_2^3 = m_2^{31} \cdot m_2^1.$$

Using (8), we receive the resultant transformation as a product of the matrices:

$$\rho[I^3] = [J^{32}] \cdot [I^2] = [J^{31}] \cdot [I^1]$$

Where matrix

$$[J^{31}] = \begin{pmatrix} m_1^{31} & 0 & 0 \\ 0 & m_2^{31} & 0 \\ \frac{1}{I_1^{G1}}(m_1^{31} - 1) & \frac{1}{I_2^{G2}}(m_2^{31} - 1) & 1 \end{pmatrix}.$$

Performance of group properties is advantage of projective transformations.

Expression (8) as projective transformation with the parameters $m_1^{21}, m_2^{21}$, translates any initial points of a plane $I_1, I_2$ in new position, how the arrows show in Fig.3. Fixed points and straight lines, which are shown by closed arrows, are visually visible.

Once again we will notice that the offered formulas of recalculation are especially convenient, when the fixed values of $m_1^{21}$, $m_2^{21}$ are used for any values of initial currents.

## IV. CONCLUSION

The projective geometry adequately interprets "kinematics" of a circuit with changeable parameters of loads, allows performing the deeper analysis. The obtained formulas of the recalculation of the currents possess the group properties at change of conductivity of the loads. It allows expressing the final values of the currents through intermediate changes of the currents and conductivities.

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