TWO – DIMENSIONAL DISCRET WAVELET TRANSFORM FOR MULTIMEDIA APPLICATIONS

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I. INTRODUCTION

In all multimedia systems a central part of the system is the block of compression algorithms responsie for an important reduction of the data amount. The compression have to be performed with a minimum numbers of errors.

Let us take a look at an illustrative example. If a video frame has a resolution of 288 by 352 (288 lines and 352 pixels per line) in CIF (Common Intermediate Format) format, each of the three primary colors (red, green and blue) is represented for one pixel with eight bits and the frame rate of transmission is 25 frames per seconds, then the bit rate is 288 x 352 x 8 x 25 = 20,275,200 bps. In a video transmission with a modem which operates at a maximum bit rate of 56,600 bits per second it is necessary to reduce the video data amount by at least 359 times. If we desire a better resolution for the video frame the amount of data grows about ten times.

All the video compression algorithms perform the transform coding in three steps. In the first step the image (frame) are divided into blocks for each component. This step is known as preprocessing operation. In the next step a particularly linear transform is performed for each block. Finally, in the last step the transformed signal is truncated, quantized and encoded.

The studies of the two-dimensional transforms presented in this paper were realized in order to obtain a clear and concise synthesis of multimedia video compression algorithms. These researches of the two-dimensional transforms used in image processing were finalized with publication of a book [8] for the students and Ph.D. students.

II. DISCRETE TWO-DIMENSIONAL WAVELET TRANSFORM

For the begining let us consider the orthogonal case and one-dimensional case. The algorithm allows a fast computation of the finite energy signals projections on different subspaces as the elements of an orthogonal multiresolution analysis [2],[10],[12].

The totality of the closed Hilbert spaces (subspaces of $L^2(R)$) $\{V_m\}_{m\in Z}$ forms a multiresolution analysis of $L^2(R)$ space if its elements satisfy the following conditions:

$$...V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 ... \tag{1}$$

$$\overline{\bigcup_{m\in Z}} V_m = L^2(\Re) \tag{2}$$

$$\bigcap_{m\in Z} V_m = \{0\} \tag{3}$$

$$(\forall) f \in V_0 \Leftrightarrow f\left(2^m t - n\right) \in V_m \tag{4}$$

and there is $\phi(t) \in V_0$ such that $\{\phi(t-n)\}_{n\in Z}$ forms a Riesz base of V_0 space (5)

The function $\phi(t)$ is named the scale function. Taking into account that $V_0 \subset V_1$, it results that there is a numbers sequence $h_k \in l^2(Z)$ such that

$$\phi(t) = 2\sum_{k\in Z} h_k \cdot \phi(2t-k) \tag{6}$$

The space W_j is built as a complement of the V_j space into V_{j+1} space and it satisfies the condition:

$$V_{j+1} = V_j \oplus W_j \tag{7}$$

Because $\psi(t)$ is an element of the V_1 space $(W_0 \subset V_1)$ and $\{\phi_{1,k}\}_{k\in Z}$ is a Riesz base of V_1, there is a sequence $g_k \in l^2(Z)$ such that:

$$\psi(t) = 2\sum_{k\in Z} g_k \cdot \phi(2t-k) \tag{8}$$

Using the definition of the multiresolution analysis it can be demonstrated that if $\{\phi(t-k)\}_{k\in Z}$ is an orthogonal base of V_0 then the functions set $\{\phi_{j,k}(t)\}_{j,k\in Z}$ defined in the same way:

$$\phi_{j,k}(t) = 2^{\frac{j}{2}} \cdot \phi\left(2^j t - k\right) \tag{9}$$

form an orthogonal base for V_j.

The same steps are performed for the wavelet function $\psi(t)$. If the function set $\{\psi(t-k)\}_{k\in Z}$ is an orthogonal base for W_0, then the functions set $\{\psi_{j,k}(t)\}_{j,k\in Z}$ defined in the same way:

$$\psi_{j,k}(t) = 2^{\frac{j}{2}} \cdot \psi\left(2^j t - k\right) \tag{10}$$

forms an orthogonal base for $L^2(R)$.

Let us denote by P_j the projection operator on V_j space and by Q_j the projection operator on W_j space. Consequently, both operators are orthogonal projection operators and that implies:

$$P_j f(t) = \sum_{k\in Z} \langle f(t), \phi_{j,k}(t) \rangle \cdot \phi_{j,k}(t) \tag{11}$$

$$Q_j f(t) = \sum_{k\in Z} \langle f(t), \psi_{j,k}(t)\rangle \cdot \psi_{j,k}(t) \tag{12}$$

Using these two last results and taking into account that $\underset{j}{\oplus} W_j = L^2(R)$ a function $f(t) \in L^2(R)$ can be written:

$$f(t) = \sum_{j\in Z} Q_j f(t) \tag{13}$$

The sequences $\{h_k\}$ and $\{g_k\}$ represent the impulse responses of two digital filters. Theirs Fourier transforms have to fulfil the following conditions:

$$\begin{cases} \left|\hat{h}(\omega)\right|^2 + \left|\hat{h}(\omega+\pi)\right|^2 = 1 \\ \left|\hat{g}(\omega)\right|^2 + \left|\hat{g}(\omega+\pi)\right|^2 = 1 \\ \hat{g}(\omega)\cdot \hat{h}^*(\omega) + \hat{g}(\omega+\pi)\cdot \hat{h}^*(\omega+\pi) = 0 \end{cases} \tag{14}$$

A particular case of the above conditions leads to a time domain relation as follows:

$$g_k = (-1)^k h_{1-k} \tag{15}$$

that is specific to the quadratic filters.

According to (12) we may write:

$$(\forall) f(t) \in L^2(\Re),\ P_j f(t) = \sum_{k=-\infty}^{+\infty} a_{j,k} \cdot \phi_{j,k}(t) \tag{16}$$

where:

$$a_{j,k} = < f(t), \phi_{j,k}(t) > \tag{17}$$

The sequence:

$$\left\{a_{j,k}\right\}_{j,k\in Z} = \left\{< f(t), \phi_{j,k}(t) >\right\}_{j,k\in Z} \tag{18}$$

will be named *the discrete approximation of* $f(t)$.

Using the decomposition relations of the signal $f(t)$ into the subspace V_{j+1}, according to the previous discussions we obtain:

$$\begin{aligned} \langle f(t), \phi_{j,n}(t)\rangle = \sqrt{2} \sum_{k=-\infty}^{+\infty} \langle \phi(t), \phi_1(t-[k-2n])\rangle \cdot \\ \cdot \langle f(t), \phi_{j+1,k}(t)\rangle \end{aligned} \tag{19}$$

According to the two-scale equation (6), the scalar products from (19) may be rewrite according to the above notations:

$$h_{k-2n} = \langle \phi(t), \phi_1(t-[k-2n])\rangle \tag{20}$$

Therefore, equation (19) becomes: $a_{j,n} = \sqrt{2} \sum_{k=-\infty}^{+\infty} h_{k-2n} \cdot a_{j+1,k}$

(21)

Denoting by $\breve{h}_{2n-k} = h_{k-2n}$ the mirror filter of h_k, then the previous relation can be rewritten as follows:

$$a_{j,n} = \sqrt{2} \cdot \sum_{k=-\infty}^{+\infty} \breve{h}_{2n-k} \cdot a_{j+1,k} \tag{22}$$

The equation (22) shows that $a_{j,n}$ can be computed from $a_{j+1,n}$ by using the $\breve{h}_n$ filter, followed by a two-factor decimation.

Similarly, the orthogonal projection of $f(t)$ function on W_j space will be computed. Denoting by $Q_j f$ the projection operator on W_j space we may write a similar relation to (16), using the $\{\psi_{j,n}(t)\}_{n \in Z}$ base on this space:

$$(\forall) f(t) \in L^2(\Re), \quad Q_j f(t) = \sum_{k=-\infty}^{+\infty} d_{j,k} \cdot \psi_{j,k}(t) \tag{23}$$

where:

$$d_{j,k} = \langle f(t), \psi_{j,k}(t) \rangle \tag{24}$$

In the same way the set:

$$d_{j,n} = \{\langle f(t), \psi_{j,n}(t) \rangle\}_{n \in Z} \tag{25}$$

is called *discrete detail signal*.

Similarly, it can be proved that:

$$d_{j,n} = \sqrt{2} \sum_{k=-\infty}^{+\infty} g_{k-2n} \cdot a_{j+1,k} \tag{26}$$

Denoting by $\breve{g}_{2n-k} = g_{k-2n}$ the mirror filter of g_k, then the previous relation can be rewritten as follows:

$$d_{j,n} = \sqrt{2} \cdot \sum_{k=-\infty}^{+\infty} \breve{g}_{2n-k} \cdot a_{j+1,k} \tag{27}$$

The equation (27) shows that $d_{j,n}$ can be computed from $a_{j+1,n}$ by using the $\breve{g}_n$ filter. Because the original signal $f(t)$ is considered at resolution $2^0 = 1$ $(j = 0)$ then $a_{0,n}$ is the discrete approximation of the original signal at resolution 1. This is exactly the discrete-time signal $f(n)$. Hence, the set of values:

$$\{a_{-J,n}, \{d_{j,n}\}_{-J \le j \le -1}\} \tag{28}$$

represents the fast discrete wavelet transform of the $a_{0,n}$ discrete signal. The values from (28) are computed as follows:

$$\begin{cases} a_{-j,n} = \sqrt{2} \cdot \sum_{k=-\infty}^{+\infty} \breve{h}_{2n-k} \cdot a_{-j-1,k}; \quad (\forall)\, j \in N \\ d_{-j,n} = \sqrt{2} \cdot \sum_{k=-\infty}^{+\infty} \breve{g}_{2n-k} \cdot a_{-j-1,k}; \quad (\forall)\, j \in N \end{cases} \tag{29}$$

In two-dimensional case three wavelet functions can be constructed:

$$\begin{cases} \psi^1(x,y) = \phi(x) \cdot \psi(y) \\ \psi^2(x,y) = \psi(x) \cdot \phi(y) \\ \psi^2(x,y) = \psi(x) \cdot \psi(y) \end{cases} \tag{30}$$

with the one-dimensional case we can obtain the three *discrete detail signals* corresponding to the three wavelet functions:

$$\begin{cases} \left\{ d^1_{j,k',k''} \right\}_{j\in Z,\,(k',k'')\in Z^2} = \left\{ < f(x,y), \phi_{j,k'}(x) \cdot \psi_{j,k''}(y) > \right\}_{j\in Z,\,(k',k'')\in Z^2} \\ \left\{ d^2_{j,k',k''} \right\}_{j\in Z,\,(k',k'')\in Z^2} = \left\{ < f(x,y), \psi_{j,k'}(x) \cdot \phi_{j,k''}(y) > \right\}_{j\in Z,\,(k',k'')\in Z^2} \\ \left\{ d^3_{j,k',k''} \right\}_{j\in Z,\,(k',k'')\in Z^2} = \left\{ < f(x,y), \psi_{j,k'}(x) \cdot \psi_{j,k''}(y) > \right\}_{j\in Z,\,(k',k'')\in Z^2} \end{cases} \tag{31}$$

In this way the collection:

$$\left\{ a_{J,k',k''}, \quad \left\{ d^1_{j,k',k''} \right\}_{-J\le j\le -1}, \quad \left\{ d^2_{j,k',k''} \right\}_{-J\le j\le -1}, \quad \left\{ d^3_{j,k',k''} \right\}_{-J\le j\le -1} \right\} \tag{32}$$

are called discrete two-dimensional wavelet transform.

In the extended form of this abstract some results of applications will be presented.

II. CONCLUSIONS

This paper is an overview of discrete wavelet transform. In the latest years spectacular results of the wavelet theory were obtained in image compression and image analysis. Some of the algorithms based on this type of analysis were included on the last generation of JPEG and MPEG standards.

V. REFERENCES

[1] **Coifman R., Meyer Y., Wickerhauser M.V.,** *"Size Properties of Wavelet Packets",* in Wavelets and Their Applications, pp. 153-178, Boston, 1992

[2] **Daubechies I.,** *Ten Lectures on Wavelet*, SIAM, Philadelphia, 1992

[3] **Isar A., Nafornişă, L.,** *"Reprezentări timp – frecvenţă"*, Editura Politehnică, Timişoara, 1998

[4] **Javerth B., Sweldens W.,** *"An Overview of wavelet based Multiresolution Analysis"*, SIAM Review, vol. 36, no. 3, pp. 377-412, September 1994

[5] **Mateescu A., Dumitriu N., Stanciu L.,** *Semnale, circuite şi sisteme*, Editura Teora, Bucureşti, 2000

[6] **Mateescu A.,Ciochină S.,Dumitriu N.,Şerbănescu A.,Stanciu L.,** *Prelucrarea numerică a semnalelor*, Editura Tehnică, Bucureşti, 1997

[7] **Poularikas A., Dorf R.C.,** The Transform and Data Compression Handbook, 2001 CRC Press LLC

[8] **Răducanu M.,** *Sisteme şi aplicaţii multimedia – Transformări biparametrice utilizate în analiza imaginilor*, Editura Electronica 2000, Bucureşti, 2004

[9] **Răducanu M.,** *Sisteme şi aplicaţii multimedia – Algoritmi de compresie pentru semnale video*, Editura MatrixRom, Bucureşti, 2004

[10] **Shi Y.Q., Sun H.,** *Image and VideoCompression for MultimediaEngineering – Fundamentals, Algorithms, and Standards*, Editura CRC şi IEEE, 2000

[12] *Wickerhauser M.V., Lectures on wavelet packets algoritms*, Tehnical Report, November 1999