ON THE INTERACTION BETWEEN THE STANDING AND TRAVELING WAVES IN THE IDEAL CONTINUOUS MEDIUM

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INTRODUCTION

In this article we continue the examination of interactions between waves in ideal medium. The sense of these examinations consists in development identical approach for description the behavior of waves, and particles. In the previous article [1] we have shown, that, the stable standing waves interact as the particles in case of elastic collision, and the conservation laws of energy and impulse follow from the undular nature of particles. Now we propose to investigate the interaction process between the stable standing wave and a traveling wave, and find the circumstances in which takes place the quantification of traveling waves.

1. WAVES AS INSTRUMENTS OF LENGTH AND TIME MEASUREMENT

Let's suppose that there is a standing wave-object (fig. 1), described in laboratory system by expression $a = A(r)\sin kr \sin \omega t$. (1)

Where *k*- is the a wave number and ω- frequency.

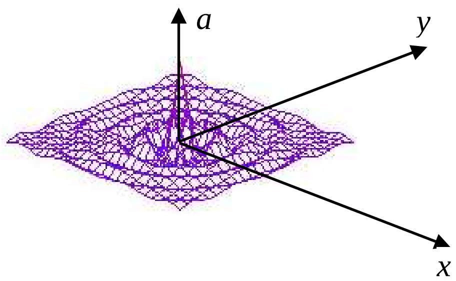

Figure 1. Wave described by expression (1) in two-dimensional representation.

The projection of the wave (1) on the axis *x* will be: $a = A(x)\sin kx \sin \omega t$. (2)

In the reference frame, which moves with velocity *v* along axis *x* relatively of laboratory system, the same wave (2) will be described by expression [2].

$$a' = A'(x)\sin\left(\frac{k_1 + k_2}{2}x' - \frac{\omega_1 - \omega_2}{2}t'\right)\times \times\sin\left(\frac{\omega_1 + \omega_2}{2}t' + \frac{k_1 - k_2}{2}x'\right). \quad (3)$$

We shall be convinced of it. If to compare (3) with (2), we see that, in (3) the value $\frac{k_1 + k_2}{2}$ have the same role as *k* in (2). Both expressions cos(*kx*) and $\cos\left(\frac{k_1 + k_2}{2}x'\right)$ describes the function fixed in space, which serve as natural unites or standards of length.

Similarly, the expressions cos(*ωt*) and $\cos\left(\frac{\omega_1 + \omega_2}{2}t'\right)$ represents the change of amplitude in a fixed point of laboratory and moving systems accordingly, in dependence from the time and serves as the time standards. Hence the value

$$\omega' = \frac{\omega_1 + \omega_2}{2}. \quad (4)$$

in moving system have the same role as *ω* in laboratory system.

The ratio of wave-object displacement, during a period to this period represents the quasi-standing wave velocity: $v = \frac{\omega_1 - \omega_2}{k_1 + k_2}$. (5)

It is easy to be convinced of it, by following directly the evolution of wave - object.

2. INTERACTION OF THE TRAVELING WAVE WITH STATIONARY STABLE WAVE-OBJECT

Now lets suppose that the wave-object is stable. It means that, the wave-object in the own reference frame remains always invariant, independent of exterior actions and is described by expression (1) and (2). The own system we call a reference frame in which $\omega_1 = \omega_2$ and $k_1 = k_2$.

The incident traveling wave having the view $a_i = A\cos(\omega_i t - k_i x)$, (6)

will try to deform a wave (1) as is shown in a fig. 2. But as, the wave-object is stable, by definition, it can not be described in the own system by other expression, than (1) or (2). Hence, the wave-object can't remain fixed in laboratory system. It will be forced to move, hence this wave in laboratory system will be described already by expression (3).

However in own system it will continue to be described by expression (1) by virtue of its stability.

As was noted above, time and lengths standards, defined by frequency and wave number, in expression (2) and (3) differ. Thus, under action of a traveling wave, the wave-object will change its standards of length and time to be adapted, to new conditions, created by presence of the incident wave. Such incompatibility of wave-object with the incident wave lead to changes of wave-object state. It means wave-object will be transferred in other frame of reference.

It is possible to assume, that the adaptation of wave-object will happen, when the change of frequency of wave-object will be equal to change of frequency of the incident wave or more simple when the change of frequency of wave-object will be equal to frequency incident wave. It means, that the incident wave does not influence wave-object, because the standards of the incident wave enter in a composition of standards of a wave-object. It actually so, if to recollect a principle of identical change of frequencies and wave numbers of interacting waves, which we have proved in the article [1], i.e. $\Delta\omega = \omega_i$. (7)

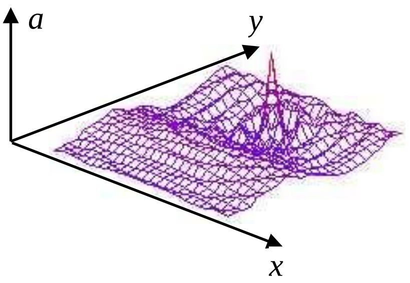

Figure 3. The wave (1), under interaction of a wave (6) in two-dimensional representation

The standing wave (2) represents a superposition of traveling waves pair from the point of view of own frame of reference. The expression (3) describes the same wave, but in a frame of reference, which moves concerning a own frame of reference with velocity *v*. The link between frequencies and wave numbers measured in these two frames, is defined by the Doppler's formulas for longitudinal effect:

$$\omega_1 = \omega\sqrt{\frac{1-\beta}{1+\beta}}, \qquad \omega_2 = \omega\sqrt{\frac{1+\beta}{1-\beta}}, \qquad (8)$$

$$k_1 = k\sqrt{\frac{1-\beta}{1+\beta}}, \qquad k_2 = k\sqrt{\frac{1+\beta}{1-\beta}}. \qquad (9)$$

Where $\beta = \frac{v}{c}$ - is the normalized velocity of relative motion, between two systems. The Doppler effect is a consequence of Lorentz transformations, hence, it is an organic part of our model, in which both, the tools and the explored objects, represents the waves in same medium [2, 3].

By substitution the expressions (8) and (9) in (4), we shall receive a relation between the oscillation frequency of wave-object ω in own system and its frequency ω', measured in stroked system which moves: $\omega' = \frac{\omega}{\sqrt{1-\beta^2}}$. (10)

If are given the frequencies of wave-object before interaction ω and after interaction ω', and the wave incident frequency ω_i, we can determinate the wave-object velocity after interaction. By substituting named values in (7), we shall receive:

$$\omega_i - \omega_i' = \omega' - \omega. \qquad (11)$$

In the case of total absorption $\omega_i' = 0$. Hence

$$\omega_i = \omega' - \omega, \qquad (12)$$

and considering (10) $\omega_i = \omega\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)$. (13)

Solving this equation, we shall receive the velocity, gained by wave-object as a result of interaction with the incident wave.

$$\beta = \pm\frac{\sqrt{\omega_i^2 + 2\omega_i\omega}}{\omega_i + \omega}. \qquad (14)$$

The same formula will be obtained in case of interaction between the quantum of light and elementary particle. Really, the law of conservation of energy at interaction of a light quantum with a fundamental particle is described by the equation:

$$\hbar\omega_i = \frac{mc^2}{\sqrt{1-\beta^2}} - mc^2,$$

solution of which is

$$\beta = \pm\frac{\sqrt{h^2\omega_i^2 + 2h\omega_i mc^2}}{h\omega_i + mc^2}.$$

This is equivalent to expression (14). By using this formula it is possible, for example, to calculate velocity, gained by the free electron after action with "light quantum" having the frequency ω_i.

Now let's consider the case, more general, when the front of an incident wave is not homogeneous. Hence, the action of incident wave is asymmetrically relatively to center of wave-object. In this case, after interaction the wave-object will move not along the axis *x*, but under some angle ψ relatively to *x*. Simultaneously should deviate and incident wave on some angle φ, by virtue of a principle of identical change of wave numbers of interacting waves, (fig. 3). In mechanics terms this case corresponds to a not central collision of two balls.

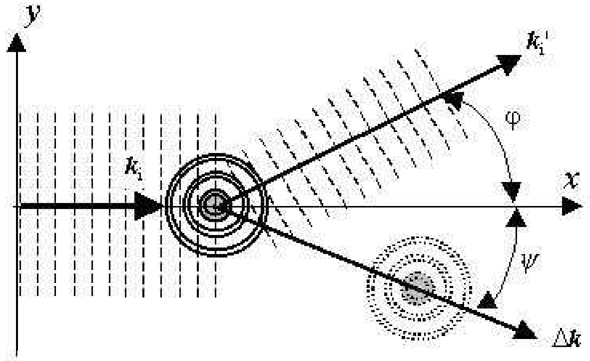

Figure 3. A wave (1), under interaction of a non uniform traveling wave.

At first we shall find out, how the frequencies and wave numbers changing at interaction. Let's suppose that, before interaction, the wave-object is resting in laboratory system and is described by expression (1). Then, before interaction, its frequency will be ω, and the resulting wave number is equal to zero $\Delta k = 0$. For incident wave (6) frequencies before interaction is equal ω_i, and its wave number

$$k_i = \frac{\omega_i}{c} = {}^{\omega_i}\!/_{c} . \qquad (15)$$

Accordingly to expression (10) the frequency of wave-object after interaction $\omega' = \frac{\omega}{\sqrt{1-\beta^2}}$.

Hence, the change of wave-object frequency as a interaction result: $\Delta\omega = \omega\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)$. (16)

Analyzing the change of wave numbers, it is necessary to mean that, it is vectors. In the own frame of wave-object, the wave numbers vectors of traveling waves-components have opposite directions and modules equal. Therefore, the resulting wave number of wave-object in state of repose $k_r = 0$. (17)

After interaction, the wave - object will gain some velocity *v*, and will be is described by expression (3) in which the modules of wave numbers of two components are not equal. The resultant wave number k_r' of wave-object is equal to half of difference of waves numbers components. In view of expression (9), the resultant wave number of moving wave-object will be:

$$k_r' = \frac{k_1 - k_2}{2} = \frac{k}{2}\left(\sqrt{\frac{1+\beta}{1-\beta}} - \sqrt{\frac{1-\beta}{1+\beta}}\right),$$

$$k_r' = \frac{\beta k}{\sqrt{1-\beta^2}} . \qquad (18)$$

Thus, as a result of interaction, wave number of wave-object will change on

$$\Delta k_r = k_r' - k_r , \qquad (19)$$

or in view of expressions (17) and (18):

$$\Delta k_r = \frac{\beta k}{\sqrt{1-\beta^2}} . \qquad (20)$$

Accordingly with the formula (15), to this value of a wave number correspond some value $\Delta\omega_r$,

$$\Delta\omega_r = \frac{\omega_1 - \omega_2}{2} = \frac{\beta\omega}{\sqrt{1-\beta^2}} , \qquad (21)$$

or, taking in account (10): $\Delta\omega_r = \omega'\beta$. (22)

Then the following difference can be conversed:

$$\omega'^2 - \omega^2 = (\omega')^2 - (\omega')^2(1-\beta^2) = (\omega')^2\beta^2 . \qquad (23)$$

From here, using the formula (22), we shall receive

$$(\omega')^2 - \omega^2 = (\Delta\omega_r)^2 . \qquad (24)$$

Now we shall analyze directly the interaction between the incident wave and the wave-object. Using the principle of identical frequency change of interacting waves, proved by us before, we can note: $\Delta\omega = -\Delta\omega_i$ (25)

or $\omega' - \omega = \omega_i - \omega_i'$ (26)

As the wave number is a vector, the principle of identical change of wave numbers of interacting waves should be noted separately for projections to each axis. This will look as follows:

$$\Delta k_r \cos\varphi = k_i - k_i' \cos\varphi \qquad (27)$$

$$\Delta k_r \sin\varphi = k_i' \sin\varphi , \qquad (28)$$

Let's raise to the second power the expressions (27), (28) and sum it, we shall receive:

$$(\Delta k_r)^2 = k_i^2 + (k_i')^2 - 2k_i k_i' \cos\varphi .$$

Dividing this expression by c^2, and taking into account the expression (15), we shall receive

$$(\Delta\omega_r)^2 = \omega_i^2 + (\omega_i')^2 - 2\omega_i\omega_i' \cos\varphi . \qquad (29)$$

From expression (26) we shall receive:

$$\begin{aligned}(\omega')^2 &= (\omega_i - \omega_i' + \omega)^2 = \\ &= \omega_i^2 + (\omega_i')^2 - 2\omega_i\omega_i' + 2\omega(\omega_i - \omega_i') + \omega^2\end{aligned} . \qquad (30)$$

Let's subtract (29) from (30):

$$\begin{aligned}(\omega')^2 - (\Delta\omega_r)^2 &= \\ &= -2\omega_i\omega_i'(1-\cos\varphi) + 2\omega(\omega_i - \omega_i') + \omega^2\end{aligned} ,$$

or

$$\begin{aligned}(\omega')^2 - (\Delta\omega_r)^2 - \omega^2 &= \\ &= -2\omega_i\omega_i'(1-\cos\varphi) + 2\omega(\omega_i - \omega_i')\end{aligned} . \qquad (31)$$

In correspondence with expression (24), the left part (31) is equal to zero, so:

$$\frac{1-\cos\varphi}{\omega}=\frac{1}{\omega_i'}-\frac{1}{\omega_i}. \qquad (32)$$

Taking into account, that $\omega=\frac{2\pi c}{\lambda}$, the expression (32) will be copied as: $\lambda_i-\lambda_i'=\lambda(1-\cos\varphi)$. (33)

Where: λ_i and λ_i' are waves length of incident and diffused traveling waves, and λ is wave length of wave-object "resting" in laboratory system. If to designate the change of length of the traveling wave, occurred as the result of its interaction with a standing wave as $\Delta\lambda_i=\lambda_i'-\lambda_i$, that we shall receive the Compton's formula:

$$\Delta\lambda_i=2\lambda\sin^2\frac{\varphi}{2}. \qquad (34)$$

The expression (34) was obtained without any suppositions, concerning the corpuscular properties of waves and without such concepts as mass, impulse or energy. By thus is shown, that the Compton's effect describe the interaction between standing wave and traveling wave.

If $\lambda_i >> \lambda$, the interaction will be practically always central. In this case the change of frequency of wave-object will be maximal and will be equal to frequency of the incident wave ω_i. Or else, the wave-object will take the greatest possible quantity of energy from the incident wave.

In all analyzed cases of interaction between the waves, the result depends only from a relation of frequencies of interacting waves and does not depend on their amplitude. That is as accepted in a quantum mechanics and proved experimentally. From the demonstrations above mentioned, is visible, that the quantification of a traveling wave, in particular of light, happens in the moment of interaction with the stable standing wave.

3. THE LOCALIZATION AND PROBABILITY OF WAVES INTERACTION

Other interesting aspect linked with interaction between the traveling wave and the stable standing wave is a problem, about where this interaction happens and also what is the probability of this interaction.

In the case of solid mechanical bodies the interaction happens in the touch point of bodies. For the waves, situation is other. At first, the interaction happens, in some volume, and in second, the interaction will be maximal effective, when the amplitude values of interacting waves will be equal. This is general requirement of the coordination between transmitter and receiver.

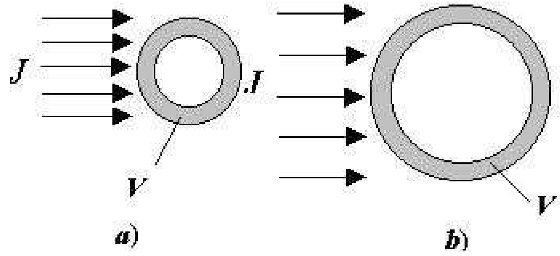

Figure 3. Domain of interaction between object and incident wave at: a) major; b) small intensity of incident wave.

As the wave-object is spherical, and incident traveling wave is flat, the position of volume, in which happens the interaction, will depend from amplitude, or intensity, of incident wave. If the incident wave has major intensity, the interaction will happen closer to center of wave-object in volume V_1 (fig. 4a), but if intensity of an incident wave is small, the wave - object "will agglomerate energy" in greater volume V_2 a (fig. 4б). The result of interaction in both cases is the same, namely, as is shown above, in correspondence with expression (25), the frequency of wave-object will vary on of incident wave frequency: $\Delta\omega=\omega_i$.

Or, if this expression multiply by Plank constant, we shall receive the known formula for energy change of wave-object at uptake of a light quantum $\Delta W=\hbar\omega_i$.

The second question consists in that, the realization of the act of interaction depends also on a relation of phases of interacting waves. In all deductions, which we have given above, we considered, that a phase of interacting waves coincide, however in practice it not always so, this fact lead to the indeterminacy in result of interaction between wave-object and incident wave.

4. INTERACTION OF TRAVELING WAVES WITH A WAVES FIELD

Above was shown that, the same laws describe the interaction between standing waves as interaction between mechanical bodies. It allows equating the stable standing waves with particles. For particles the relation between the kinetic energy W_c, the impulse p and the frequency ω is determined by the formulas:

$$W_c=W_T-W_0=m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}}-1\right),$$

$$W_c = \hbar\omega_0\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right), \qquad (64)$$

$$p = \frac{W_0}{c^2}\frac{1}{\sqrt{1-\beta^2}}v = \frac{\hbar\omega_0}{c^2}\frac{v}{\sqrt{1-\beta^2}} = \\ = \frac{\hbar}{c}\frac{\beta\omega_0}{\sqrt{1-\beta^2}} = \frac{m_0 c\beta}{\sqrt{1-\beta^2}}. \qquad (65)$$

Here:

W_0 - energy of the resting particle; W_T - total energy of a particle; p - impulse of a particle; ω_0 - frequency of a particle in own frame of reference; m_0 - mass of the resting particle.

From this it is possible to make a conclusion that, the stationary waves-objects have energy, impulse, mass and velocity. But, in this case, the problem appears with treatment of concept of mass and velocity in relation to traveling waves. Is generally accepted to consider that, the traveling waves have only energy and impulse, but it haven't the masses and it is impossible to connect with them the change of velocity concerning a frame of reference, as they are propagating with a constant value of velocity c.

The concept of mass follows from the Newton's second law. Mass represent a resistance of the body to force, which acts on the body and changes its velocity. If we refuse to waves travelling in property of mass, by thus we consider that, they can affect particles (or standing waves), but on them are impossible to act. This contradicts the third law Newton's, relativity principle and at last, to common sense.

In the article [4] we have offered to introduce the concept of inertness. It allows applying the identical approach for travelling and standing waves. Let's define inertness m as the ratio of wave energy to a quadrate of traveling waves velocity propagation. In our case this is light velocity. Then, according to this definition, inertness will have both, standing and traveling waves.

The application of inertness concept for light waves will allow us to express losses of energy by a wave not only through change of its frequency. The change of waves energy can be expressed through the change of observer velocity v with help of the Doppler's formulas. Such approach is expansion of the relativity principle. Really, if we can speak about change of wave energy (or frequency) at change of the observer velocity, that, with the same basis we can speak that, the frequency has varied because there is equivalent change of wave velocity. In correspondence with the relativity principle, the results of observations should not depend on the one who changes the velocity: the observer, or observed object. Thus, we introduce the concept of change of equivalent wave velocity v_E, and we accord to traveling wave the status of object.

We can formulate the following definition. The equivalent change of wave velocity Δv is equal to such change of the observer velocity, which calls the same change of wave frequency at the expense of Doppler effect. Naturally, the absolute equivalent velocity not exists, there is only relative change of velocity at transition from one frame to another.

The change of equivalent wave velocity Δv should not be confused with velocity of wave propagation c, which remains always constant in all frames, or with oscillatory velocity, caused by wave in medium, known in acoustics.

The concepts of inertness and equivalent velocity allow viewing interactions of all types of waves from positions unified. In particular, it is possible to execute the analysis of interaction of a traveling wave with a random undular field representing a superposition of a great many of chaotically propagating traveling waves.

The travelling wave represents the directional transfer of energy and impulse in medium. If medium is homogeneous, the motion from one layer will be transmitted completely to other layer, just as the motion between two identical billiard balls is transmitted. If the balls are not identical, the motion from one ball to another ball will not be transmitted completely. Similarly, if the layers of medium differ from each other by the parameters, the transmission of energy will be not complete. The wave in this case will lose the energy, or we can say the wave will transmit energy of heterogeneity.

Any wave, in itself, creates in medium the heterogeneity, therefore, the wave transiting through a field of other waves, will lead to partial losing of its energy.

In acoustics this fact usually is ignored, as losses at the expense of a dissipation of energy in acoustic mediums is usually significant more, than losses at the expense of interaction with other waves.

In ideal medium, the losses of wave energy will be only at the expense of transmission of energy to other waves, which form a chaotic undular field. Thus, the losses of energy (and consequently also change of frequency) of examined wave will be proportional to the effective value of excess density of medium created by the undular field.

As was shown above, **as a result of waves interaction their frequency varies, but not their intensity**. Is natural to extending this fact to interaction between travelling waves with the arbitrary undular field. The interest represents a case, when the light wave comes from the spatial objects situated very far. In this case, light wave on its trajectory will interact with other light fields for a long time, and the part of energy, which will be transmitted to a chaotic undular field, existing in space, becomes considerable.

It is known that light, which comes from the far spatial objects experience so-called red shift of spectral lines. It means spectrum of radiation is displaced in the party of lower frequencies. Now this change of a spectrum is attributed to a Doppler's effect. It is considered that, the Universe is expanding, and as the object is further situated, so faster it moves off us. One of problems of this hypothesis consist in that, irrespective of where is situated the observer, for him the "expanding" Universe looks like so, as if he is in centre, whence expansion began. It is so-called model of the inflationary Universe (or a pudding model - Universe is represented as a pudding, which grows on yeast).

We propose more simple and natural explanation of red shift, namely: red shift of light, which comes from the far spatial objects, is caused by its interaction with other undular fields. In this case, the fact why the red shift is proportional to distance up to a light source becomes natural and clear.

Let's assume that, the basic part of such interaction happens to so-called relict radiation, existing in space. Knowing density of relict radiation and frequency change of light on unit of length, we can estimate vacuum density.

Let's make this estimation. We consider, that the ratio of density change of radiation energy ΔE on unity of length to density of energy of radiation E is equal to the ratio of a medial additional effective value of density of vacuum, created by oscillations of relict radiation $\Delta\rho$, to density of empty space in a unperturbed state ρ_0:

$$\frac{\Delta E}{E} = \frac{\Delta\rho}{\rho_0}. \tag{40}$$

By multiplication on the numerator and denominator of the right fraction on c^2, we shall receive:

$$\frac{\Delta E}{E} = \frac{w}{\rho_0 c^2}. \tag{41}$$

Here w is density of energy of relict radiation.

The inertness of radiation $m = E/c^2$, as was spotted above. Hence, the losses of energy density of radiation per unit of length

$$\Delta E = m\Delta v^2 = E\frac{\Delta v^2}{c^2},$$

where Δv - equivalent change of wave velocity of radiation from the far space object per unit of length. Thus

$$\frac{\Delta E}{E} = \frac{\Delta v^2}{c^2}. \tag{42}$$

Combining the expressions (41) and (42) we shall receive:

$$\frac{\Delta v^2}{c^2} = \frac{w}{\rho_0 c^2}. \tag{43}$$

Hence

$$\rho_0 = \frac{w}{\Delta v^2}.$$

It is possible to express Δv through the Hubble's constant H as $\Delta v = HR$,
where R represents the distance, on which there was a change of light energy w. Then the final formula will look as.

$$\rho_0 = \frac{w}{(HR)^2} \tag{44}$$

Let's substitute the following numerical values in (44): $w = 6\times10^{-14}$ J/m^3, $H = 2{,}44\times10^{-18}$ 1/s, $R = 1$ m.
We receive the value $\boldsymbol{\rho_0 \geq 1\times10^{22}}$ kg/m^3
The shown method allows only estimating the order of vacuum density. This method does not take into account many factors. The Hubble's constant considers the losses of energy at the expense of all interactions, to which are undergone the light. The share of relict radiation in these losses can be much less. And the denominator should take into account only this share. Therefore it is necessary to expect, that the obtained value for ρ_0 represent the minimal limit of vacuum density. The really value can be much more.

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