INFLUENCE OF THE THERMOVISCOPLASTIC PROPERTIES OF THE STRUCTURAL MODEL SUBELEMENTS ON THE THERMOVISCOELASTIC CHARACTERISTICS OF MATERIAL

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INTRODUCTION

Every material, being isotropic and homogeneous at the macroscopic level, may be represented as a conglomerate composed of an infinite number of kinematically connected elementary compositions of the material particles, which are grouped according to a common parameter governing the considered phenomenon. Structural elements possess only simple properties, but in result of their interaction at macroscopic scale one may describe very complex phenomena.

In the investigated model functions, reflecting thermoviscoplastic properties of subelements, are regarded as depending on the rate of deformation change of the body element, which in turn influences the thermoviscoelastic characteristics of material. The kinematic coherence of subelements leads to the interdependence of phenomena of the different nature.

1. BASIC EQUATIONS OF THE STRUCTURAL MODEL

The macroscopic element, being at the original time in natural state, is subjected to mechanical and thermal action. It is assumed that during deformation the material behavior depends significantly on the rate of loading and heating. To describe the microheterogeneous medium behavior the macroscopically homogeneous volume element V_0 of the polycrystalline body is considered to be composed of an infinite number of kinematically connected subelements with different thermorheological features. These subelements are successively involved in a process of irreversible deformation.

Subelement is identified with the set of all material particles inside the conglomerate V_0 that have the same irreversible strain tensors

$$\overline{p}_{ij} = \tilde{p}_{ij}, \qquad \overline{p}_{ij} = \left\langle \tilde{p}_{ij} \right\rangle_{\overline{V}}, \tag{1}$$

where $\overline{p}_{ij}$ implies average irreversible strain of the subelement with the volume $\overline{V}$.

Particles of the same subelement may have different orientations and situations in the conglomerate space. Because the granules of the polycrystalline aggregate are nonuniformly deformed so that the mass and the volume of a single accepted subelement can be arbitrarily small. It is evident that proceeding from the selection of material particles in the accordance with the irreversible strain tensor other thermo-mechanical quantities change from a material particle to other one in the given subelement.

Let us represent the stress and strain tensors of the macroelement of the microheterogeneous aggregate as a sum of deviators and spherical tensors:

$$t_{ij} = \sigma_{ij} + \sigma_0 \delta_{ij}, \qquad \sigma_0 = t_{ii}/3, \tag{2}$$

$$d_{ij} = \varepsilon_{ij} + \varepsilon_0 \delta_{ij}, \qquad \varepsilon_0 = d_{ii}/3. \tag{3}$$

Components of the strain and stress deviators of the volume element are the weighted means of the strains and stresses deviators of the subelements

$$\sigma_{ij} = \int_0^1 \overline{\sigma}_{ij} d\psi, \qquad \varepsilon_{ij} = \int_0^1 \overline{\varepsilon}_{ij} d\psi, \tag{4}$$

where as the state parameter, identifying quantities $\overline{\sigma}_{ij}$ and $\overline{\varepsilon}_{ij}$ with certain subelement, is chosen weight of the irreversibly deformed subelements $\psi\ (0 \le \psi \le 1)$, reflecting the sequence of the subelements transition from the reversible to irreversible state under the initial loading.

Describing inelastic behavior of polycrystalline aggregate, it's very important to evaluate the influence of the heterogeneous distribution of irreversible deformations in the interior domain V_0 on the macroscopic relationships between stresses and strains. Therefore the local interconnection in the examined model is established between reversible and irreversible deformations:

$$\overline{\varepsilon}_{ij} = \overline{e}_{ij} + \overline{p}_{ij}, \tag{5}$$

$$\varepsilon_{ij} = e_{ij} + p_{ij}, \qquad \varepsilon_0 = (e_0 + \varepsilon_T) + p_0. \tag{6}$$

Rate of change of the irreversible deformation path length is the state parameter that reflects the sensitivity of subelement to rate of external action's change:

$$\dot{\lambda} = \sqrt{\dot{\bar{p}}_{ij}\dot{\bar{p}}_{ij}}\,. \tag{7}$$

Evolutional equation of the state parameter $\bar{s}$, characterizing the isotropic hardening owing to the modification of structure in the irreversible processes, is accepted as

$$\dot{\bar{s}} = \begin{cases} a\dot{\bar{\lambda}}, & \bar{s} < \bar{x}(\gamma,\upsilon), \\ \dot{\bar{x}}, & \bar{s} = \bar{x}(\gamma,\upsilon). \end{cases} \tag{8}$$

At the beginning of the irreversible deformation process $\bar{s}\big|_{t=0} = s_0$, where s_0 depends on the type of the heat treating of the material. If at the start of the process of the irreversible deformation the material is in the structurally stable state, then $s_0 = 0$.

The relation between the kinematic hardening $\bar{r}$ and the state parameters is expressed as follows

$$\bar{r} = \begin{cases} a_0\bar{p}, & a_0\bar{p} < \bar{x}_0(\gamma,\upsilon), \\ \bar{x}_0(\gamma,\upsilon), & a_0\bar{p} = \bar{x}_0(\gamma,\upsilon), \end{cases} \tag{9}$$

$$\bar{r} = \sqrt{\bar{r}_{ij}\bar{r}_{ij}}\,, \qquad \bar{r}_{ij} = \bar{r}\,\frac{\bar{p}_{ij}}{\bar{p}}\,. \tag{10}$$

Kinematic relations and local physical laws of deformation are given within the limits of the examined structural model assuming that all types of interactions between subelements in the conglomerate are formed only under the influence of average connections, i.e. material particles in the aggregate do not deform independently, but only in a coordinated manner.

The interaction between two subelements is realized by means of the interactions between material particles which are appertained to the different subelements. This fact is reflected by replacement of the local state parameters in physical equation for subelement on the average values of the whole set:

$$\gamma = \frac{1}{\psi_\lambda}\int_0^1 \dot{\bar{\lambda}}(\psi')d\psi', \qquad \upsilon = \frac{1}{\psi_\upsilon}\int_0^1 \bar{\upsilon}(\psi')d\psi'\,, \tag{11}$$

$$\dot{s} = \frac{1}{\psi_s}\int_0^1 \dot{\bar{s}}(\psi')d\psi'\,, \qquad 0 \le \psi_\lambda, \psi_\upsilon, \psi_s \le 1\,, \tag{12}$$

where γ is the average rate of change of the irreversible strain in the subset of subelements, being under the loading above the elastic limit; υ describes inelastic volume variation; ψ is the distinctive parameter of subelement which during the initial loading coincides with the weight of irreversibly deformed subelements when this subelement exceeded the elastic limit; ψ_λ, ψ_υ, ψ_s – summary weights of subelements for which the corresponding parameters $\dot{\bar{\lambda}}$, $\bar{\upsilon}$, $\dot{\bar{s}}$ are nonzero.

The state parameter υ describes rheological effects of the subelement, and is expressed by the ratio of the volume variation and its limit possible value, being the same for all subelements

$$\upsilon = \frac{1}{\varepsilon_{0k}}\int_0^1 (\bar{\varepsilon}_0 - \beta\bar{e}_0)d\psi\,, \tag{13}$$

or taking into consideration that

$$\varepsilon_0 = \int_0^1 \bar{\varepsilon}_0 d\psi\,, \qquad e_0 = \int_0^1 \bar{e}_0 d\psi\,, \tag{14}$$

we obtain

$$\upsilon = \frac{\varepsilon_0 - \beta(\upsilon)\dfrac{\sigma_0}{K(\upsilon)}}{\varepsilon_{0k}}\,, \tag{15}$$

where K is the volume compressibility modulus.

Differentiating (15) with respect to time we find the loading conditions at $\upsilon = const$:

$$\dot{\varepsilon}_0 = \beta\dot{e}_0 = \beta\frac{\dot{\sigma}_0}{K}\,. \tag{16}$$

As was demonstrated in [9], for $\beta = 1$ the deformation process at $\upsilon = const$ corresponds to isothermal loading.

In monotonous processes throughout the subset of irreversibly deformed subelements only active processes of loading occur, that corresponds to the monotony of the evolution of weight of irreversibly deformed subelements in these processes. This means that towards ψ the single separation boundary forms between reversibly $\psi' < \psi \le 1$ and irreversibly $0 \le \psi \le \psi'$ deformed subelements. The variations $d\bar{p}$ in all irreversibly deformed subelements have one and the same sign.

The phenomenon of the auto concordance of irreversible strain processes of subelements can be represented according to the concept of the average connections in the way of two equations [2, 6], which can be written in the case of a monotonous process as follows:

– the yield condition for the subelement under the influence of structural modifications in conglomerate

$$\bar{e}_{ij}\frac{dp_{ij}}{d\lambda}=\tau(\psi,\gamma,\upsilon)+s+\bar{r}\cos\alpha\,, \qquad (17)$$

$$d\bar{p}\big|_{\psi>\psi'}=0\,, \quad d\lambda=\int_0^{\psi'}d\bar{\lambda}d\psi\,, \qquad (18)$$

$$s\big|_{\bar{s}<\bar{x}}=a\lambda\,, \quad \gamma=\frac{\dot{\lambda}}{\psi'}\,; \qquad (19)$$

– the law about the general orientation of the irreversible yield processes in subelements

$$\frac{d\bar{p}}{d\bar{\lambda}}=\frac{dp}{d\lambda} \quad \text{or} \quad \frac{d\bar{p}_{ij}}{d\bar{\lambda}}=\frac{dp_{ij}}{d\lambda}\,, \qquad (20)$$

that means

$$\cos\bar{\alpha}=\frac{\bar{p}_{ij}}{\bar{p}}\frac{d\bar{p}_{ij}}{d\bar{\lambda}}=\frac{p_{ij}}{p}\frac{dp_{ij}}{d\lambda}=\cos\alpha\,, \qquad (21)$$

where the functional τ represents scalar properties of subelements in the structurally stable state and can be identified with the initial yield point of subelement; $\bar{\alpha}$ is the angle between the tangent to the irreversible strain path and irreversible strain vector.

Equations of the kinematic connection between subelements, satisfying the first law of thermodynamics, were obtained in [4,6,7] on the basis of the stresses and strains fluctuations principle, formulated by V. Marina [3]:

$$\Delta\bar{t}_{ij}=-B\Delta\bar{d}_{ij}+\alpha\sqrt{\frac{B(B+K)}{3}\Delta\bar{d}_{nm}\Delta\bar{d}_{nm}}\;\delta_{ij}\,, \qquad (22)$$

$$\alpha=\begin{cases}1, & dacă \quad \bar{d}_{nm}\bar{d}_{nm}>d_{pq}d_{pq}\\ -1, & dacă \quad \bar{d}_{nm}\bar{d}_{nm}\le d_{pq}d_{pq}\end{cases},$$

$$\bar{t}_{ij}=t_{ij}+\Delta\bar{t}_{ij}\,, \quad \bar{d}_{ij}=d_{ij}+\Delta\bar{d}_{ij}\,, \qquad (23)$$

$$\left\langle\Delta\bar{t}_{ij}\right\rangle=0\,, \quad \left\langle\Delta\bar{d}_{ij}\right\rangle=0\,, \quad \left\langle\Delta\bar{t}_{ij}\Delta\bar{d}_{ij}\right\rangle=0\,, \qquad (24)$$

where $\Delta\bar{t}_{ij}$, $\Delta\bar{d}_{ij}$ are stresses and strains fluctuations; K is the volume compressibility modulus; B is the internal parameter, reflecting simultaneously the heterogeneity of the processes of deformation and loading of subelements in the conglomerate.

In consequence of the decomposition of stresses and strains fluctuations into the deviators and the spherical tensors

$$\Delta\bar{t}_{ij}=\Delta\bar{\sigma}_{ij}+\Delta\bar{\sigma}_0\delta_{ij}\,, \quad \Delta\bar{d}_{ij}=\Delta\bar{\varepsilon}_{ij}+\Delta\bar{\varepsilon}_0\delta_{ij}\,, \qquad (25)$$

two groups of equations are obtained

$$\Delta\bar{\sigma}_{ij}=-B\Delta\bar{\varepsilon}_{ij}\,, \qquad (26)$$

$$\Delta\bar{\sigma}_0=\alpha\sqrt{\frac{BK}{3}\Delta\bar{\varepsilon}_{nm}\Delta\bar{\varepsilon}_{nm}}\,, \qquad (27)$$

$$\alpha=\begin{cases}1, & dacă \quad \bar{\varepsilon}_{nm}\bar{\varepsilon}_{nm}>\varepsilon_{pq}\varepsilon_{pq}\\ -1, & dacă \quad \bar{\varepsilon}_{nm}\bar{\varepsilon}_{nm}\le\varepsilon_{pq}\varepsilon_{pq}\end{cases}.$$

Assuming that the elastic properties of subelements and of the body element are identical

$$\bar{e}_{ij}=\frac{\bar{\sigma}_{ij}}{2G(\gamma,\upsilon)}\,, \quad e_{ij}=\frac{\sigma_{ij}}{2G(\gamma,\upsilon)}\,, \qquad (28)$$

the equations of fluctuations of reversible and irreversible strains are obtained from (26)

$$\bar{e}_{ij}-e_{ij}=m\left(p_{ij}-\bar{p}_{ij}\right),\quad m=\frac{B}{B+2G}\,. \qquad (29)$$

Unknown internal parameter m is determined on the basis of the principle of the measures discrepancy, formulated by V.Marina [3-4]: in all real interactions in conglomerate the discrepancy between the macroscopic measure and the suitable microscopic analogue reaches extreme values

$$\left\langle\bar{\sigma}_{ij}\bar{\varepsilon}_{ij}\right\rangle-\left\langle\bar{\sigma}_{ij}\right\rangle\left\langle\bar{\varepsilon}_{ij}\right\rangle=Extr\,. \qquad (30)$$

Parameter of the kinematic scheme m, as follows from extremum of the discrepancy Δ [6,8], depends on the linear hardening coefficient a_0:

$$m=-a_0+\sqrt{a_0+a_0^{\;2}}\,. \qquad (31)$$

The internal parameter B is determined from the relation (29):

$$B(\gamma,\upsilon)=2G(\gamma,\upsilon)\frac{m}{1-m}\,. \qquad (32)$$

2. PROPORTIONAL LOADING

We will examine the deformation of the body element along a rectilinear trajectory to determine the rheological functions $\tau=\tau(\psi,\gamma,\upsilon)$, reflecting thermoviscoplastic properties of the subelements.

Tensor properties of subelements in conglomerate under proportional loading are given, taking into account that the directrixes of the deviators of reversible e_{ij}, $\bar{e}_{ij}$ and irreversible p_{ij}, $\bar{p}_{ij}$ strains coincide:

$$\frac{\bar{e}_{ij}}{\bar{e}}=\frac{e_{ij}}{e}=\frac{\bar{p}_{ij}}{\bar{p}}=\frac{p_{ij}}{p}=a_{ij}\,, \qquad (33)$$

$$\bar{e} = \sqrt{\bar{e}_{ij}\bar{e}_{ij}} \ , \quad \bar{p} = \sqrt{\bar{p}_{ij}\bar{p}_{ij}} \ . \qquad (34)$$

Inserting (28) and (6) into (33) we find

$$\frac{\sigma_{ij}}{\sigma} = \frac{\varepsilon_{ij}}{\varepsilon} = a_{ij} \ , \qquad (35)$$

$$\sigma = \sqrt{\sigma_{ij}\sigma_{ij}} \ , \qquad \varepsilon = \sqrt{\varepsilon_{ij}\varepsilon_{ij}} \ . \qquad (36)$$

The local relation between reversible and irreversible strains (29) is represented in the form:

$$\bar{e} - e = m(p - \bar{p}). \qquad (37)$$

In the monotonous process the irreversible deforming law (17) for the group of the irreversibly deformed subelements $0 \le \psi \le \psi'$ is written as:

$$\bar{e} = \tau(\psi, \gamma, \upsilon) + s + \bar{r} \ , \qquad (38)$$

where

$$\cos\alpha = \frac{p_{ij}}{p}\frac{dp_{ij}}{d\lambda} = 1 , \qquad s = ap \ , \qquad \bar{r} = a_0\bar{p} \ . \quad (39)$$

According to (37), for the group of subelements, operating in the reversible domain ($\psi' < \psi \le 1$, $\bar{p} = 0$), the elastic strains of sub-elements are identical and coincide with the limit elastic strain in the boundary subelement $\psi = \psi'$:

$$\bar{e} = e + mp = \tau(\psi', \gamma, \upsilon) + s \ . \qquad (40)$$

The function $\tau(\psi', \gamma, \upsilon)$, reflecting thermo-viscoplastic properties of the subelements without regard the structure evolution, can be expressed in terms of macroscopic quantities:

$$\tau(\psi', \gamma, \upsilon) = e + (m - a)p \ . \qquad (41)$$

Differentiating (37) and (38) at a constant values of the state parameters γ and υ, we can conclude that the rate of change of the irreversible deformation has the same value for all of the subelements from the subset $\psi \le \psi'$

$$\dot{\bar{p}} = \frac{\dot{e} + (m - a)\dot{p}}{a_0 + m} , \qquad (42)$$

$$\dot{e} = \frac{d}{dt}\sqrt{e_{ij}e_{ij}} \ , \qquad \dot{p} = \frac{d}{dt}\sqrt{p_{ij}p_{ij}} \ . \qquad (43)$$

According to the average connections concept

$$\dot{p} = \int_0^1 \dot{\bar{p}}d\psi = \int_0^{\psi'} \dot{\bar{p}}d\psi + \int_{\psi'}^1 \dot{\bar{p}}d\psi = \int_0^{\psi'} \dot{\bar{p}}d\psi \ . \quad (44)$$

Thus the distinctive parameter of subelements ψ' can be represented by the following relation

$$\psi' = \frac{\dot{p}(a_0 + m)}{\dot{e} + (m - a)\dot{p}} . \qquad (45)$$

In the monotonous process of deformation along a rectilinear path from (7) and (33)-(34)

$$\dot{\bar{\lambda}} = \frac{d}{dt}\sqrt{\bar{p}_{ij}\bar{p}_{ij}} = \dot{\bar{p}} \ . \qquad (46)$$

Substituting (46) into (11) and taking into account that $\dot{\bar{p}}$ has the same value for all of the irreversibly deformed subelements $\psi \le \psi'$

$$\gamma = \frac{1}{\psi'}\int_0^{\psi'} \dot{\bar{p}}d\psi = \dot{\bar{p}} \ , \qquad \dot{\bar{p}}\Big|_{\psi > \psi'} = 0 \ . \qquad (47)$$

Inserting (42) into (47), the state parameter γ is expressed as

$$\gamma = \frac{\dot{e} + (m - a)\dot{p}}{a_0 + m} . \qquad (48)$$

The structure of the relation (48) follows that within the limits of the investigated model the condition of continuity of the material transition from reversible to irreversible state is satisfied automatically.

Indeed, we note that at the initial moment of the flow $\dot{p} = 0$ so the parameter γ is proportional to rate of the total strain change $\dot{\varepsilon} = \dot{e}$:

$$\gamma = \frac{\dot{\varepsilon}}{a_0 + m} . \qquad (49)$$

Therefore the considered model does not provoke any uncertainties in the moment of the material transition from reversible to irreversible state.

Let's pass in (48) to rates of change of the modules of the stress and strains deviators

$$\gamma = \frac{1 - m + a}{a_0 + m}\frac{\dot{\sigma}}{2G} + \frac{m - a}{a_0 + m}\dot{\varepsilon} \ , \qquad (50)$$

where

$$\dot{\varepsilon} = \frac{d}{dt}\sqrt{\varepsilon_{ij}\varepsilon_{ij}} \ , \qquad \dot{\sigma} = \frac{d}{dt}\sqrt{\sigma_{ij}\sigma_{ij}} \ . \quad (51)$$

We'll examine the isothermal loading of the thin-walled tubes with the tensile force F and the internal pressure P_i. The radial stress t_{rr}, being of order of internal pressure, can be neglected in comparison to the axial t_{zz} and circumferential $t_{\varphi\varphi}$ stresses

$$t_{zz} = \frac{F}{2\pi Rh} + \frac{P_i R}{2h} , \qquad t_{\varphi\varphi} = \frac{P_i R}{h} . \qquad (52)$$

The orientation of the loading path in the space of axial t_{zz} and circumferential $t_{\varphi\varphi}$ stresses we'll define by the parameter

$$\zeta = \frac{t_{\varphi\varphi}}{t_{zz}}, \tag{53}$$

and determine the ratio of tensile force and the internal pressure

$$\frac{F}{P_i} = \frac{2-\zeta}{\zeta}\pi R^2. \tag{54}$$

In [10] it was established that in laboratory conditions the simplest experiment at the constant parameter γ is the experiment at the constant velocity of the gripping device motion $\dot{d}_{zz}$, if the orientation of rectilinear path of loading in the space of stresses t_{zz} and $t_{\varphi\varphi}$ is specified as follows:

$$\zeta = \frac{2-\eta}{1+\eta}, \qquad \eta = \beta\frac{2G}{K}\frac{m-a}{1-m+a}. \tag{55}$$

The stress-strain diagram obtained by isothermal loading at $\dot{d}_{zz} = const$ is plotted in the space of the deviator modules of reversible and reversible strains $e \sim p$ (figure 1)

$$\sigma = \sqrt{\frac{2}{3}\left(\zeta^2 - \zeta + 1\right)}\,t_{zz}, \quad \varepsilon = \sqrt{\frac{3}{2}}\left(d_{zz} - \varepsilon_0\right), \tag{56}$$

$$e = \sqrt{\frac{2}{3}\left(\zeta^2 - \zeta + 1\right)}\frac{t_{zz}}{2G}, \qquad p = \varepsilon - e, \tag{57}$$

where d_{zz} is axial strain of the thin-walled sample.

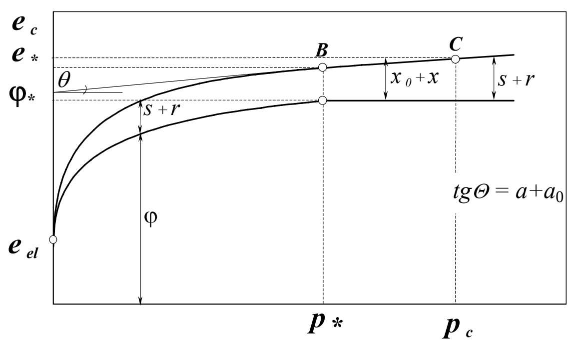


Figure 1. Diagram of the proportional loading of the thin-walled tubes by axial force and internal pressure at the constant temperature and the rate of change of the axial strain.

Taking into consideration that at $\varepsilon = \varepsilon_{el}$ is still satisfied the relation

$$\varepsilon_0 = \frac{1-2\nu}{3}d_{zz}, \tag{58}$$

we can write the expression for the parameter γ (49) in this way

$$\gamma = \sqrt{\frac{2}{3}}\frac{1+\nu}{a_0+m}\dot{d}_{zz}, \tag{59}$$

where ν is Poisson's ratio, $\dot{d}_{zz}$ is the velocity of the gripping device motion.

By means of the diagram $e \sim p$, obtained under isothermal proportional loading, the thermo-viscoplastic properties of subelements can be determined at $\gamma = const$, $\upsilon = const$:

$$e = \varphi(p,\gamma,\upsilon) + s + r. \tag{60}$$

During the scleronomous and isothermal loading we have $e,_p = \varphi,_p + a + a_0$, hence the relations (41) and (45) are obtained as follows

$$\tau(\psi',\gamma,\upsilon) = \varphi(p,\gamma,\upsilon) + (m + a_0)p, \tag{61}$$

$$\psi' = \frac{a_0+m}{\varphi,_p + m + a_0}. \tag{62}$$

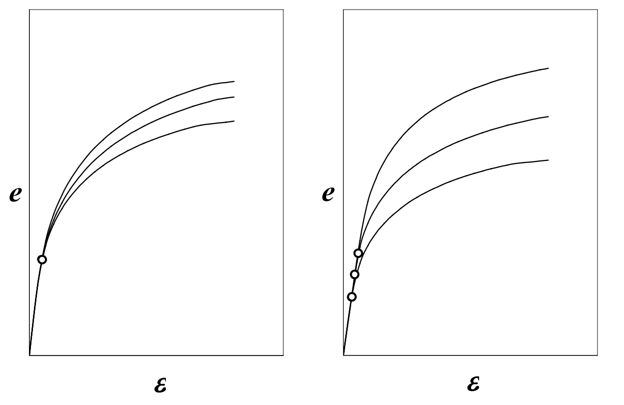
Thus, the thermoviscoplastic properties of subelements are determined on the basis of the proportional loading diagrams $e = e(p,\gamma,\upsilon)$ at the various constant value of the state parameters γ and υ.

In the investigated model it's considered that rheological functions $\tau(\psi',\gamma,\upsilon)$, reflecting thermoviscoplastic properties of subelements without regard the structure evolution, depend on the average rate of change of irreversible deformations γ. In the classical theories, conversely, it is assumed that rate of change of irreversible deformation depends on the stress. In the case of a single-element model both concepts lead to the same results. However, for a conglomerate, consisting of a finite or infinite number of subelements, these concepts lead to different results in describing the behavior of the material.

The functional $\tau(\psi',\gamma,\upsilon)$ represents the limiting elastic strain in the structurally stable state (without hardening). As long as all subelements in the volume element V_0 are in the reversible state, the weight of irreversibly subelements stressed beyond the limit of elasticity $\psi' = 0$. Taking into account (41), (61) and (49), we obtain

$$\tau(0,\dot{\varepsilon},\upsilon)=\varphi(0,\dot{\varepsilon},\upsilon)=\varepsilon_{el}(\dot{\varepsilon},\upsilon). \qquad (63)$$

The classical theories follow that the elastic limit of the conglomerate does not depend on the strain rate $\dot{\varepsilon}$ (figure 2).



a) **b)**

Figure 2. Influence of the strain rate $\dot{\varepsilon}$ on the elastic limit of the conglomerate: ***a)*** *in the classical theories;* ***b)*** *in the investigated model.*

3. THERMOVISCOELASTIC PROCESSES

It's possible to distinguish three stages of deformation during monotonic loading of the system of subelements. At an initial stage, all subelements operate in the elastic domain, because the processes, occurring under the condition $\varepsilon \le \varepsilon_{el}(\gamma,\upsilon)$, will be called thermoviscoelastic. If $\varepsilon_{el}(\gamma,\upsilon)<\varepsilon<\varepsilon_*(\gamma,\upsilon)$, then the part of the subelements is in the reversible state and the other in irreversible. Such processes can be called thermoviscoelasticplastic. In the third stage of the conglomerate deformation all subelements operate beyond the limit of elasticity $\varepsilon>\varepsilon_*(\gamma,\upsilon)$. In this case it is about the thermoviscoplastic processes. Further we'll analyze in detail the thermoviscoelastic process.

In the thermoviscoelastic process all subelements are in the reversible state, so in the volume element V_0 of the polycrystalline body the weight of irreversibly subelements stressed beyond the limit of elasticity $\psi'=0$.

Under proportional loading tensor properties of subelements in conglomerate are given by (35), taking into consideration that the directrixes of the deviators of stresses σ_{ij} and total strains ε_{ij} coincide (35):

$$\sigma_{ij}=\frac{\sigma}{\varepsilon}\varepsilon_{ij}. \qquad (64)$$

In this case

$$\sigma=2G(\gamma,\upsilon)\varepsilon\le\sigma_{el}=2G(\gamma,\upsilon)\varepsilon_{el}(\gamma,\upsilon). \quad (65)$$

The volume compressibility modulus K is assumed independent of rate of strain change, so the following relationship between spherical tensors of stresses and strains is satisfied

$$\sigma_0=K(\upsilon)(\varepsilon_0-\varepsilon_T), \qquad (66)$$

where ε_0 is the total volume modification; ε_T – non-mechanical volume change (thermal $\alpha_T T$, structural, etc.).

Taking into account (65) and (66), Poisson's ratio is determined by the formula

$$\nu(\gamma,\upsilon)=\frac{K(\upsilon)-2G(\gamma,\upsilon)}{2[K(\upsilon)+G(\gamma,\upsilon)]}=\frac{1}{2}\left(1-\frac{E(\gamma,\upsilon)}{K(\upsilon)}\right). \quad (67)$$

From (67) it follows that if shear modulus G increases then Poisson's ratio ν is reduced. This phenomenon has wide experimental confirmation [1, 5, etc.].

According to (65) the thermoviscoelastic processes are determined by two factors: the relationship between shear modulus G and state parameters γ and υ, as well as the extension of the thermoviscoelastic state of the body element $\varepsilon_{el}(\gamma,\upsilon)=\tau(0,\gamma,\upsilon)$.

Duration of the reversible state with respect to the deformation is determined not thermoviscoelastic properties, defined by function $G=G(\gamma,\upsilon)$, but by thermoviscoplastic properties defined by function $\varepsilon_{el}=\varepsilon_{el}(\gamma,\upsilon)$. This phenomenon is not peculiar to subelements, taken separately, and follows from the kinematic coherence of system of subelements. As a result, despite the fact that the deformation of the volume element starts at zero rate of change of macroscopic deformation $\dot{p}=0$, but in the weakest subelement $\dot{\bar{p}}>0$ therefore, proceeding from (48), we obtain for the variable state parameters γ and υ

$$\gamma=\dot{\bar{p}}=\frac{\dot{\varepsilon}-\varepsilon_{el,\gamma}\dot{\gamma}-\varepsilon_{el,\upsilon}\dot{\upsilon}}{m+a_0}. \qquad (68)$$

As a result of this the elastic limits of subelements $\tau(\psi,\gamma,\upsilon)$ become dependent on the rate of deformation change of the body element, which in turn leads to a change in the value ε_{el}. Thus, the thermoviscoplastic properties of the subelements due to the continuity of the material transition from reversible to irreversible state influence thermoviscoelastic characteristics of material. The kinematic coherence of subelements leads to the interdependence of phenomena of the different nature.

To determine the duration of the reversible deformation of the body element we use the condition of continuity of material transition from reversible to irreversible state. At the beginning of the micro flow $t = t_1$, taking into account that $\dot{p}(t_1) = 0$

$$\gamma(t_1) = \frac{\dot{\varepsilon}(t_1) - \varepsilon_{el,\upsilon}\dot{\upsilon}(t_1)}{m + a_0}, \qquad \dot{\gamma}(t_1) = 0. \qquad (69)$$

Taking into account (69) in the condition $\varepsilon \le \varepsilon_{el}(\gamma, \upsilon)$ we obtain the equation to determine moment of occurrence of flow

$$\varepsilon_{el}\left(\frac{\dot{\varepsilon}(t_1) - \varepsilon_{el,\upsilon}\dot{\upsilon}(t_1)}{m + a_0}, \upsilon(t_1)\right) = \varepsilon(t_1). \qquad (70)$$

Equations (69), (70) are convenient to use when histories of change of deformation $\varepsilon = \varepsilon(t)$ and temperature $\upsilon = \upsilon(t)$ are defined.

The viscoelastic properties affect only the length of reversible state, if the shear modulus $G(\upsilon)$ is independent of the loading rate. In the case when histories of the loading $\sigma = \sigma(t)$ and heating $\upsilon = \upsilon(t)$ are given, expressions (69) and (70) can be represented in the form

$$\gamma(t_1) = \frac{1}{m + a_0}\left[\frac{\dot{\sigma}(t_1)}{2G} - \left(\frac{\sigma(t_1)G_{,\upsilon}}{2G^2} + \varepsilon_{el,\upsilon}\right)\dot{\upsilon}(t_1)\right], \quad (71)$$

$$\varepsilon_{el}\left(\frac{\frac{\dot{\sigma}(t_1)}{2G} - \left(\frac{\sigma(t_1)G_{,\upsilon}}{2G^2} + \varepsilon_{el,\upsilon}\right)\dot{\upsilon}(t_1)}{m + a_0}, \upsilon(t_1)\right) = \varepsilon(t_1). \quad (72)$$

System (69) and (70) naturally reflects the phenomenon of delay of yield, which follows from the concept of continuity of the material transition from reversible to irreversible state. Based on this concept is possible to describe a series of thermoviscoelastic effects on the level of the body element, endowing subelements only thermoviscoplastic properties. Also it automatically removes problem of the boundary between the thermoviscoelastic and thermoviscoplastic properties of the material.

If the shear modulus *G* depends on the rate of deformation and loading, it is necessary to concretize structure of the state parameter γ in the thermoviscoelastic domain. We assume that the expression for γ (50), obtained from the analysis of behavior of system of subelements in irreversible domain, remains unchanged under reversible loading.

Under proportional loading the system of constitutive equations is written as:

$$\sigma = 2G(\gamma, \upsilon)\varepsilon, \qquad \varepsilon \le \varepsilon_{el}(\gamma, \upsilon), \qquad (73)$$

$$\gamma = \frac{1 - m + a}{a + m}\frac{\dot{\sigma}}{2G} + \frac{m - a}{a + m}\dot{\varepsilon}. \qquad (74)$$

Independence the stress deviator from the deformation path in the reversible domain is shown in figure 3.

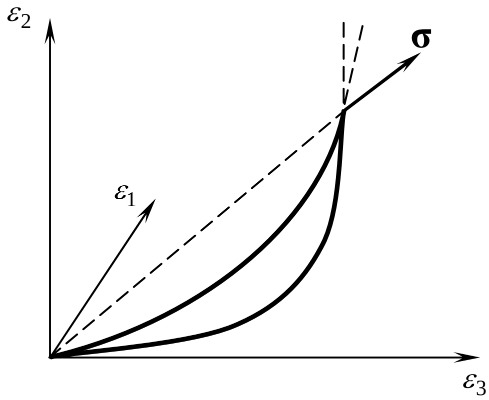


Figure 3. Deformation path in the space of the total strains.

Integrating (74) for zero initial conditions, we calculate the variation of stress (strain) for a given process of change of temperature and strain (stress).

If during loading the strain exceeds the value $\varepsilon_{el}(\gamma, \upsilon)$, then irreversible deformations occur in the macroelement. The relationship $\sigma \sim \varepsilon$ is complicated and modification of the system of the constitutive equations (73)-(74) becomes invalid.

CONCLUSIONS

In the investigated model it's considered that the elastic limits of subelements $\tau(\psi', \gamma, \upsilon)$ depend on the rate of deformation change of the body element γ, which in turn leads to a change in the value ε_{el}. Thus, the thermoviscoplastic properties of the subelements, due to the continuity of the material transition from reversible to irreversible state, that the structure of the relation (48) confirms, influence thermoviscoelastic characteristics of material. The kinematic coherence of subelements leads to the interdependence of phenomena of the different nature.

In the classical theories, conversely, it is assumed that rate of change of irreversible deformation depends on the stress. In the case of a single-element model both concepts lead to the same results. However, for a conglomerate, consisting of a finite or infinite number of subelements, these concepts lead to different results in describing the behavior of the material. The classical theories follow that the elastic limit of the conglomerate does not depend on the strain rate $\dot{\varepsilon}$.

The resulting equations of thermoviscoelastic process (73)-(74) describe the characteristic features of material in reversible domain. Under loading at constant rate of strain change $\dot{\varepsilon} = const$ this system follows as $\dot{\sigma} = const$ and vice versa, which results to a linear relationship between stress and strain in such tests. System (73)-(74) are describing the creep in the experiments when $\sigma = const$ and stress relaxation of the body element when $\varepsilon = const$.

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