# The group of $c$-reflective subcategories

BOTNARU Dumitru

In the subcategory $C_2\mathcal{V}$, of topological vector locally convex spaces Hausdorff, $\mathbb{R}_c$ is the lattice of subcategories $c$-reflective.

**Definition 1.** *A reflective subcategory it's called c-reflective, if it contains a subcategory $\mathcal{S}$ of spaces with weak topology and reflector functor $r : C_2\mathcal{V} \to \mathcal{R}$ is exactly to the left.*

**Theorem 1.** *For any subcategory $\mathcal{R}$, which contains the subcategory $\mathcal{S}$, there is the largest subcategory c-reflective $\bar{\mathcal{R}}$, what is contained in $\mathcal{R}$.*

Any $c$-reflective subcategory $\mathcal{L}$ defines a pair of conjugate subcategories $(\mathcal{K}, \mathcal{L})$, where $\mathcal{K}$ is a coreflective subcategory [1].

Let $(\mathcal{K}, \mathcal{L})$ and $(\mathcal{F}, \mathcal{R})$ be two pairs of conjugated subcategories, $\mathcal{T} = \mathcal{L} \cap \mathcal{R}$ and $\mathcal{U} = sup(\mathcal{K}, \mathcal{F})$, where the supreme is examined in the class of the coreflective subcategories of the category $C_2\mathcal{V}$.

For $\mathcal{L}$, $\mathcal{R} \in \mathbb{R}_c$ let $\rho(\mathcal{L}, \mathcal{R})$ be the full subcategory of all objects of category $C_2\mathcal{V}$ for which $\mathcal{L}$- and $\mathcal{R}$-replicas coincide.

**Theorem.** *Let $\mathcal{L}$, $\mathcal{R} \in \mathbb{R}_c$ be. Then*

*(1) $\rho(\mathcal{L}, \mathcal{R})$ is a reflective subcategory that contains the subcategory $\mathcal{S}$.*

*(2) $\rho(\mathcal{L}, \mathcal{R})$ is a $\mathcal{T}$- semireflexive subcategory [2].*

*(3) $\rho(\mathcal{L}, \mathcal{R})$ is a $\mathcal{T}$- semireflexive subcategory [2].*

**Theorem.** *Let $\mathcal{L}$, $\mathcal{R} \in \mathbb{R}_c$ be. The binary operation $\mathcal{L} \oplus \mathcal{R} = \overline{\rho(\mathcal{L}, \mathcal{R})}$ possess the following properties:*

*(1) $\oplus$ is a commutative operation.*

*(2) $C_2\mathcal{V}$ is a neutral element: $\rho(C_2\mathcal{V}, \mathcal{L}) = \mathcal{L} = C_2\mathcal{V} \oplus \mathcal{L}$ .*

*(3) Each element coincides with its neutral: $\rho(\mathcal{L}, \mathcal{L}) = C_2\mathcal{V} = \mathcal{L} \oplus \mathcal{L}$.*

# References

[1] Botnaru D., *Couples des sous-catégories conjugées*, ROMAI J. 14(2018), 1, 23-41.

[2] Botnaru D., *Groupoïd des sous-catégories $\mathcal{L}$-semi-réflexives*, Rev. Roumaine Math. Pures Appl., 63(2018), 1, 61-71.

[3] Dumitru Botnaru, Alina Ţurcanu, *The factorization of the right product of two subcategories*, ROMAI Journal, VI (2010), 2, 41-53.

(BOTNARU Dumitru)
*E-mail address*: dumitru.botnaru@gmail.com