Journal of Engineering Science | | *Vol. XXVII, no. 4 (2020), pp. 28 - 36*
Fascicle | *Industrial Engineering* | *ISSN 2587-3474*
Topic | *Mechanical Engineering and Technologies* | *eISSN 2587-3482*

DOI: 10.5281/zenodo.4296187
CZU 621.833:519.8

THE MATHEMATICAL MODEL OF THE PRECESSIONAL GEAR AND THE PROCESS OF GENERATION OF INCLINED TEETH

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Received: 10. 23. 2020
Accepted: 11. 27. 2020

Abstract. The paper deals with the development of precessional gears with toothed gearing $A_{CV-CV}^{D,\beta}$ with concave-concave contact of inclined teeth. In order to increase the bearing capacity and the mechanical efficiency of the precessional gear, the contact of the teeth is concave-concave with little difference of the curvature radius of the conjugated profiles, and to reduce the frictional sliding in the contact the teeth are inclined. In order to make the gearing $A_{CV-CV}^{D,\beta}$ with inclined teeth, there was developed the process of generating the inclined teeth $G_{m.ax}^{cil,\beta}$ with cylindrical tool on numerically controlled multi-axial machine tools.

Keywords: *precessional transmissions, concave-concave contact of inclined teeth, mathematical modeling of tooth gearing, conjugated profiles, generation of wheels on numerically controlled machine tools.*

1. The mathematical model of the gear $A_{CV-CV}^{D,\beta}$

The elaboration of the mathematical model of the gear $A^{D,\beta}$ with the gearing $A_{CV-CV}^{D,\beta}$ with concave-concave contact of the inclined teeth is based, analogous to the one with straight teeth, on the observance of the main statement of the fundamental law of gearing on ensuring continuity and constant permanent ratio of motion transformation [1].

For the elaboration of the mathematical model of the gear $A^{D,\beta}$ presented in Figure 1 we admit the following considerations, conditions and constraints [2]:

1. We admit the angle of inclination of the teeth β_g and the active length of the flanks of the conjugated teeth b_w;

2. We consider that the profile of the tooth of the satellite wheel inclined with the angle β_g, in the middle section of radius R_m is described by the circle arc LEM of radius r_m with the origin at point G_m, and the plane of the circle arc P_m is perpendicular to the line OG_m (Figure 1).

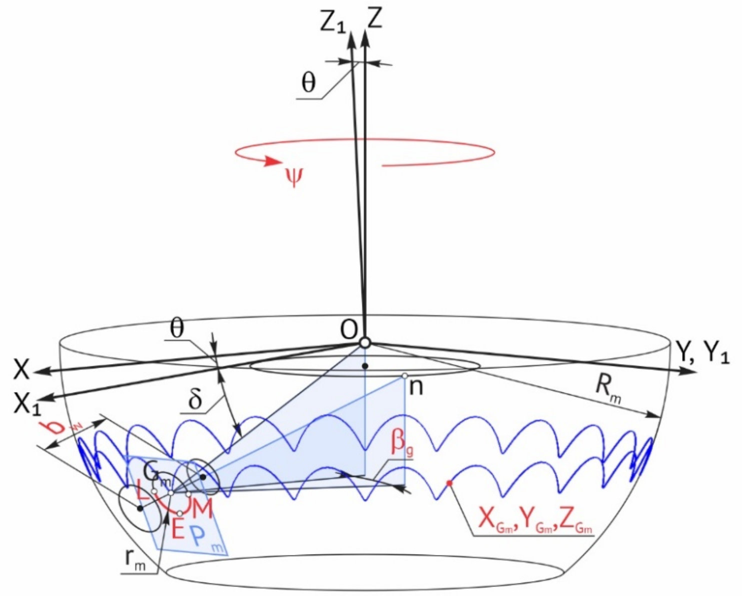

Figure 1. Description of the spatial positioning of the inclined tooth in the gear $A^{D,\beta}$.

In the spherospatial motion of the satellite wheel, the origin G_m of the radii of curvature r_m of its teeth at a precession cycle $0 \le \psi \le \dfrac{2\pi Z_2}{Z_1}$ describes on the sphere of the radius R_m a trajectory with the coordinates $X_{G_m} Y_{G_m} Z_{G_m}$.

3. The flank surface of the satellite wheel tooth represents the lateral surface of an inclined cone trunk with the extensions of the generators passing through the circle described by the arc LEM and the tip located at point n tangent to the cylinder with the Z axis of the radius e of the line $G_m n$ located in the plane P_c tangent to the conical axoid with the Z axis and the tip in the center O with the tip angle $\pi - 2(\theta + \delta)$.

$$\begin{aligned} X_G &= R\cos\delta\left[-\cos\psi\sin\left(Z_1\psi/Z_2\right)+\sin\psi\cos\left(Z_1\psi/Z_2\right)\cos\theta\right]-R\sin\delta\sin\psi\sin\theta, \\ Y_G &= -R\cos\delta\left[\sin\psi\sin\left(Z_1\psi/Z_2\right)+\cos\psi\cos\left(Z_1\psi/Z_2\right)\cos\theta\right]+R\sin\delta\cos\psi\sin\theta, \\ Z_G &= -R\cos\delta\cos\left(Z_1\psi/Z_2\right)\sin\theta-R\sin\delta\cos\theta. \end{aligned} \tag{1}$$

4. The inclination angle β_g of the satellite wheel tooth is equal to the angle between the lines OG_m and $G_m n$ projected in the plane perpendicular to the Z axis of the satellite wheel.

5. The flank surface of the central wheel tooth represents the imprint of the winding of the family of the surface of the satellite wheel tooth, which satisfies the condition $Z_{E_m} < Z_{G_m}$ of varying the angles of positioning the crankshaft ψ and of the rotation $\varphi = -\psi Z_1/Z_2$ of the satellite wheel itself relative to the crankshaft at a precession cycle [3].

Taking into account the statement in p. 2 of the geometric model of the gearing $A^{D,\beta}_{CV-CV}$, according to which the circle arc LEM in the middle section of the satellite wheel tooth is in a plane perpendicular to OG_m, then for this section the profile of the inclined teeth can be expressed by the coordinates $X_{E_m} Y_{E_m} Z_{E_m}$ similarly to the gear A^{D}_{CV-CV} with straight teeth. Figure 2 shows the profilogram of the central wheel tooth in the middle section with the radius R_m, obtained with the winding of the family of circles of radius *r* of the wheel-satellite teeth with the origins located on the trajectory of the spherospatial motion of their origin G_m.

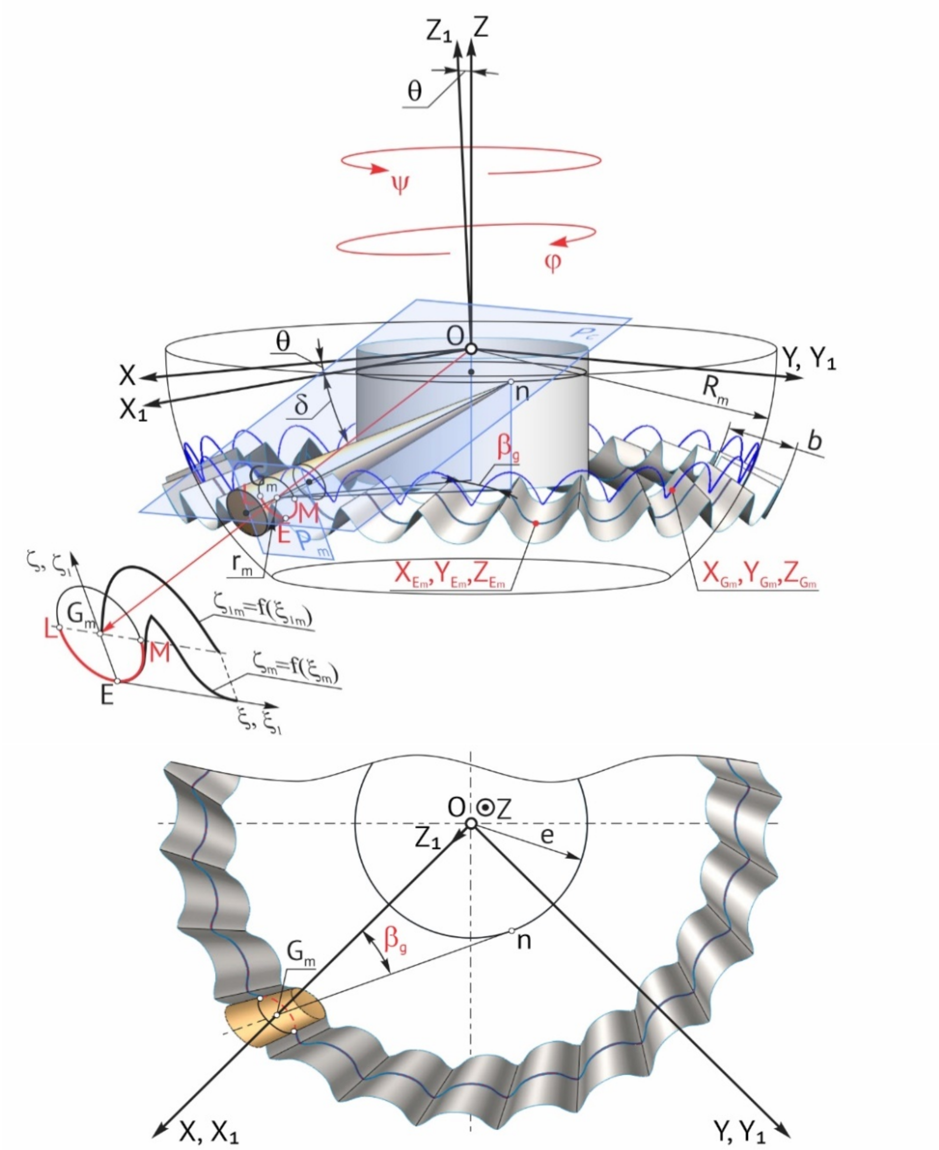

Figure 2. Description of the profile of the teeth in circle arc of the satellite wheel and covex-concave of the central wheel in the middle section.

At the same time, the profile of the central wheel teeth in the middle section R_m represents the trajectory of the contact point of the teeth E_m at a precession cycle $0 \le \psi \le \frac{2\pi Z_2}{Z_1}$ and can be expressed by the coordinates $X_{E_m} Y_{E_m} Z_{E_m}$. From Figure 2 we find that the angle between the position vector OG_m of the origin of the radius of curvature of the circle arc profile of the satellite wheel tooth G_m and the position vector OE_m of the point of contact of the teeth in the middle section with the radius R_m, represents the imaginary taper angle β of satellite wheel teeth with a circle arc profile, which can be expressed by the expression

$$\overline{OG_m} \times \overline{OE_m} = R_m^2 \cos\beta \tag{2}$$

or

$$X_{Em} \cdot X_{Gm} + Y_{Em}Y_{Gm} + Z_{Em} \cdot Z_{Gm} - R_m^2 \cos\beta = O \quad (3)$$

of which the coordinate X_{E_m}

$$X_{Em} = (R_m^2 \cos\beta - Y_{Em}Y_{Gm} - Z_{Em}Z_{Gm}) / X_{Gm} . \quad (4)$$

Substituting (4) in (3), we obtain

$$Y_{Em} = k_1 Z_{Em} - d_1 \quad (5)$$

and, replacing (5) in (4) the coordinate X_{E_m} of the contact point of the teeth in the middle section is determined from the expression:

$$X_{Em} = k_2 Z_{Em} - d_2 , \quad (6)$$

where

$$k_1 = \frac{X_{G_m}\left(X_{G_m}\dot{X}_{G_m} + Y_{G_m}\dot{Y}_{G_m}\right) + Z_{G_m}^2 \dot{X}_{G_m}}{\left(X_{G_m}\dot{Y}_{G_m} - Y_{G_m}\dot{X}_{G_m}\right) Z_{G_m}},$$

$$d_1 = R^2 \cos\beta \dot{X}_{Gm} / (X_{Gm}\dot{Y}_{Gm} - Y_{Gm}\dot{X}_{Gm}),$$

$$k_2 = -(k_1 Y_{Gm} + Z_{Gm}) / X_{Gm} , \quad (7)$$

$$d_2 = (R^2 \cos\beta + d_1 Y_{Gm}) X_{Gm}$$

The contact point of the teeth E_m also belongs, at the same time, to the sphere with radius R_m, i.e. its coordinates satisfy its equation:

$$X_{E_m}^2 + Y_{E_m}^2 + Z_{E_m}^2 - R_m^2 = 0 \quad (8)$$

Substituting (5) and (6) in (8) and solving the equation obtained in relation to the coordinate Z_{E_m} of the contact point E_m we obtain:

$$Z_{E_m} = \frac{\left(k_1 d_1 - k_2 d_2\right) \pm \left[\left(k_1 d_1 - k_2 d_2\right)^2 + \left(k_1^2 + k_2^2 + 1\right)\left(R^2 - d_1^2 - d_2^2\right)\right]^{1/2}}{k_1^2 + k_2^2 + 1} \quad (9)$$

It should be noted that the curve of the profile of the teeth of the central wheel is equidistant from the trajectory of the motion of the origin G_m of the radius of the circle arc, and for any angle of rotation ψ of the crankshaft the condition $Z_{E_m} < Z_{G_m}$ must be met.

After some transformations of the expression (9) the coordinate Z_{E_m} can be determined by the relation:

$$Z_{E_m} = \frac{\left(k_1 d_1 - k_2 d_2\right) - \left[\left(k_1 d_1 - k_2 d_2\right)^2 + \left(k_1^2 + k_2^2 + 1\right)\left(R^2 - d_1^2 - d_2^2\right)\right]^{1/2}}{k_1^2 + k_2^2 + 1} \quad (10)$$

Based on equations (5), (6) and (10) on the sphere of radius R_m the trajectory of the contact point $E_m(\psi)$ of the conjugated teeth in the gearing $A_{CV-CV}^{D,\beta}$ was interpreted depending on the precession angle ψ shown in Figure 2. The trajectory of the motion of the contact point of the conjugated teeth $E_m(\psi)$ represents the profile of the central wheel teeth in the middle section of radius R_m [4].

Applying the design procedures from spherical trigonometry, similar to the gear with straight teeth, we determine the functions $\zeta_m = f(\xi_m)$ which represents the profile of the inclined teeth in the middle section with radius R_m in plane Cartesian coordinates and the function $\zeta_{1m} = f(\xi_{1m})$ – which represents the motion trajectory of the origin of the curve of the circle arc in the middle section of the inclined tooth (see Figure 2).

The profile of the central wheel teeth presented by the function $\zeta_m = f(\xi_m)$ is characteristic only for the median section with radius R_m of the inclined teeth. It should be noted that in any other sections along the tooth of the central wheel (see Figure 3) its geometric shape in cross sections are not similar to each other (concurrent) as in the case of straight teeth. From the analysis of Figure 3 we find that in the case of teeth inclined at the angle β_g for $R_i < R_m$ the cross sections of the tooth of the central wheel, in relation to the section with radius R_m are continuously decreasing, for $R_i > R_m$ – the cross sections are continuously increasing, and their geometric shape differs from section to section.

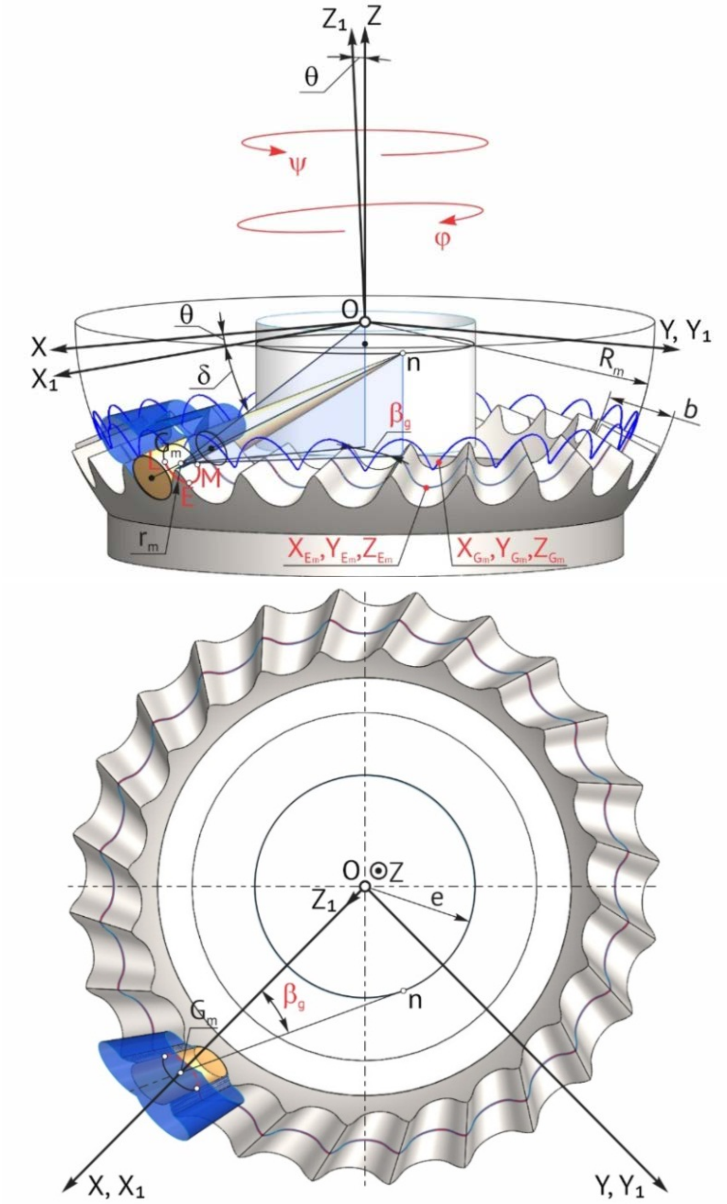

Figure 3. Geometry of the flank surface of the inclined teeth of the central wheel.

The shape of the flank profiles of the inclined tooth of the central wheel, depending on the radius R_m differs from each other with the influence on them of the taper angle β of the satellite wheel teeth calculated from the expression

$$\beta_i = arctg\frac{r}{R_i} \tag{11}$$

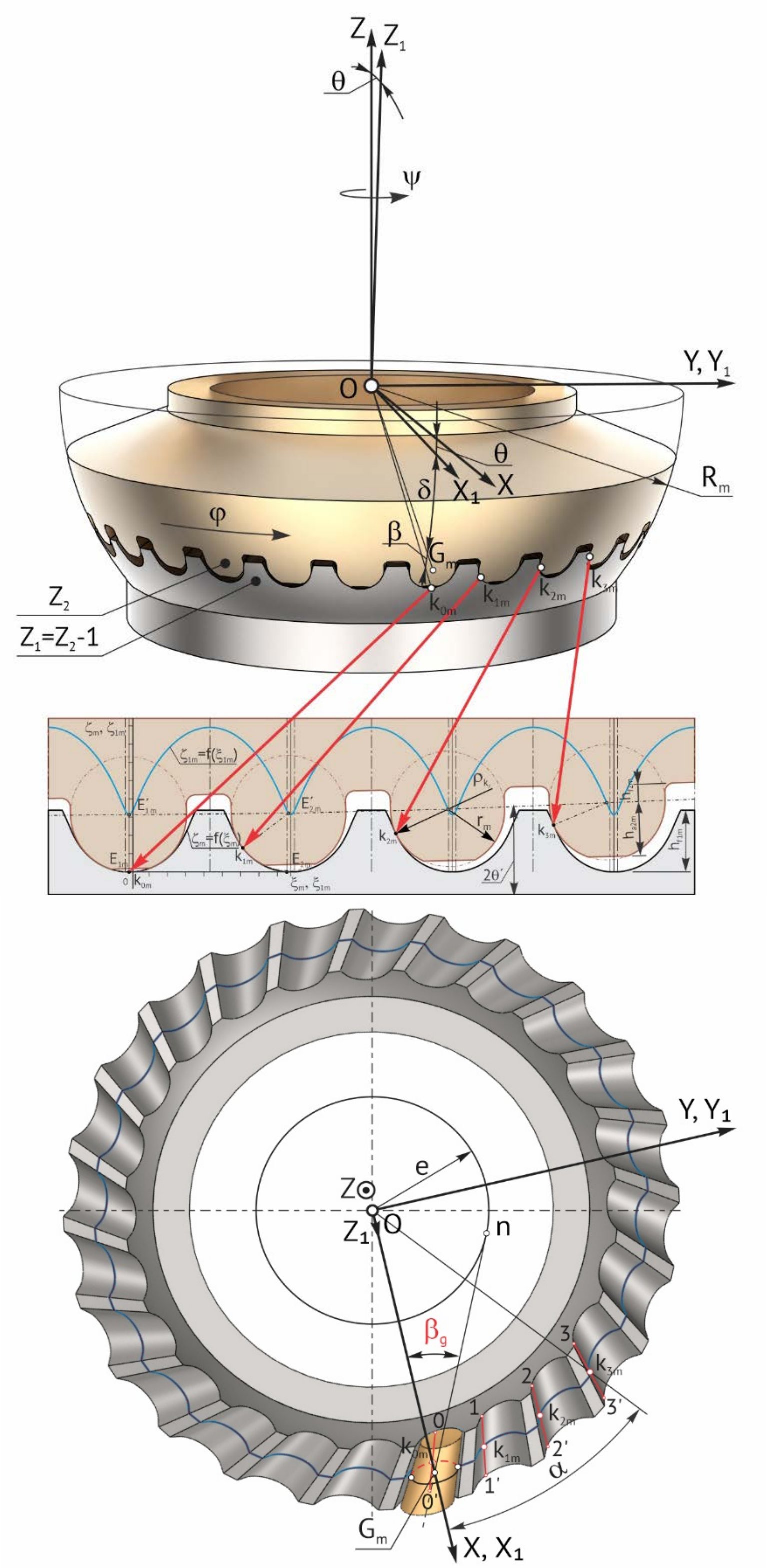

Figure 4. Evolution of the variation of the summary contact lines of the inclined teeth.

Figure 4 shows the length, variation and positioning of the contact lines of the inclined teeth engaged within the limits of the coverage area, which extends to the center angle α. From the analysis of the succession of entry and exit of tooth pairs in the gear area we find that the degree of coverage of the teeth in the gearing and the summary length of the contact lines of the geared teeth depends on the frontal coverage, determined by the multiplicity of gearing ε_f and longitudinal coverage ε_a depending on the angle of inclination of the teeth β_g, including the configuration parameters $[Z_g\text{-}\theta, \pm 1]$ and the change in the height of the teeth [5]. It is also observed that the contact lines between the inclined teeth are positioned in space so that their extensions are tangent to the cylinder with radius e.

It should be noted that the inclination of the teeth leads to a decrease in frictional sliding in the contact of the geared teeth, because the conjugation of the teeth takes place by rolling depending on the angle θ.

The profile of the inclined teeth of the central wheels in the middle section with radius R_m is considered as the basis for the elaboration of the process for generating the inclined teeth $G_{m.ax}^{D,\beta}$ on numerically controlled multi-axial machine tools [2].

2. Generation $G_{m.ax}^{cil,\beta}$ of inclined teeth with conform contact on numerically controlled multi-axial machine tools

The synthesis of the toothed precessional gear A^D with straight teeth was based on the observance of certain geometric and kinematic conditions. One of the restrictive geometric conditions is to ensure the intersection of the extensions of the flank surface generators of the teeth of the conjugated wheels at the tip point of the conical axoids, which coincides with the precession center O. This condition defines the geometry of the straight teeth.

Considering this common condition for gears A^D and A^B, in all elaborated generation processes $G_{r.s}^{con}$, $G_{r.s}^{disc}$ and $G_{m.ax}^{cil}$ the condition that the extension of the tool contact lines with the blank must pass through the precession center O is complied with. In this case the teeth generated by the generating contour of the tool will be straight.

The precessional gear $A^{D,\beta}$ with the gearing $A_{CV-CV}^{D,\beta}$ with small difference of the radii of curvature of the conjugated profiles defined by the geometry of the contact of the flanks of the teeth as concave-concave gear can be made by the process $G_{CV-CV}^{cil,\beta}$ with the cylindrical tool on numerically controlled multi-axial machine tools taking into account the peculiarities of the geometry of the conjugated teeth with the angle of inclination β_g.

The CAD/CAM design and fabrication stages of precessional teeth with inclined teeth are analogous to those of straight-toothed teeth provided that the geometric peculiarities representing the inclination of the teeth at the angle β_g, shown in Figure 2, are considered.

The essential difference between the processes $G_{m.ax}^{cil}$ and $G_{m.ax}^{cil,\beta}$ of the generation of the flanks of the teeth with the gears A_{CV-CV}^{D} and $A_{CV-CV}^{D,\beta}$ consists in the fact that in the case of straight teeth the extension of the tool contact line and the profile of the generated tooth passes through the precession center O, and in the case of inclined teeth, the generating contour of the cylindrical tool must ensure the generation of the tooth flanks according to p. 5 of the description of the mathematical model of the gear $A_{CV-CV}^{D,\beta}$.

For the elaboration of the process of generating the wheels of the gear $A^{D,\beta}$ with the gearing $A^{D,\beta}_{CV-CV}$ resented in Figures 1 and 2 we define the shape and the positioning of the tool in relation to the blank by the following geometric and kinematic parameters [6]:

1. The geometric shape of the conjugated teeth in the gearing $A^{D,\beta}_{CV-CV}$ and their positioning in space defined by the provisions p. 1 - p. 5 presented above, represent the obligatory premises of the generation process;

2. The cylindrical tool has a radius r_s that does not exceed the radius of the bottom curvature of the teeth in the section with the radius R_i of the tooth of the central wheel;

3. The numerically controlled multi-axial machine tool communicates to the tool motions in relation to the blank, so that the generating contour of the cylindrical tool with radius r_s ensures the generation according to p. 5 of the description of the mathematical model of the gear $A^{D,\beta}_{CV-CV}$.

For roughing, the deep machining method was chosen.

The deep machining method bends to the geometry of the part and has the advantage of using the active part of the cutting tool to a large extent.

The machining method involves dividing the tooth machining into two parts: the upper part and the lower part of the tooth. The height of each part is 10 mm and each part is processed using the deep machining method [3].

For semi-finishing, the 3D machining method was chosen with constant steps - which involves traversing the surface of the teeth by the cutting tool - spherical milling cutter with two teeth and radius, for example 6 mm - with the same value.

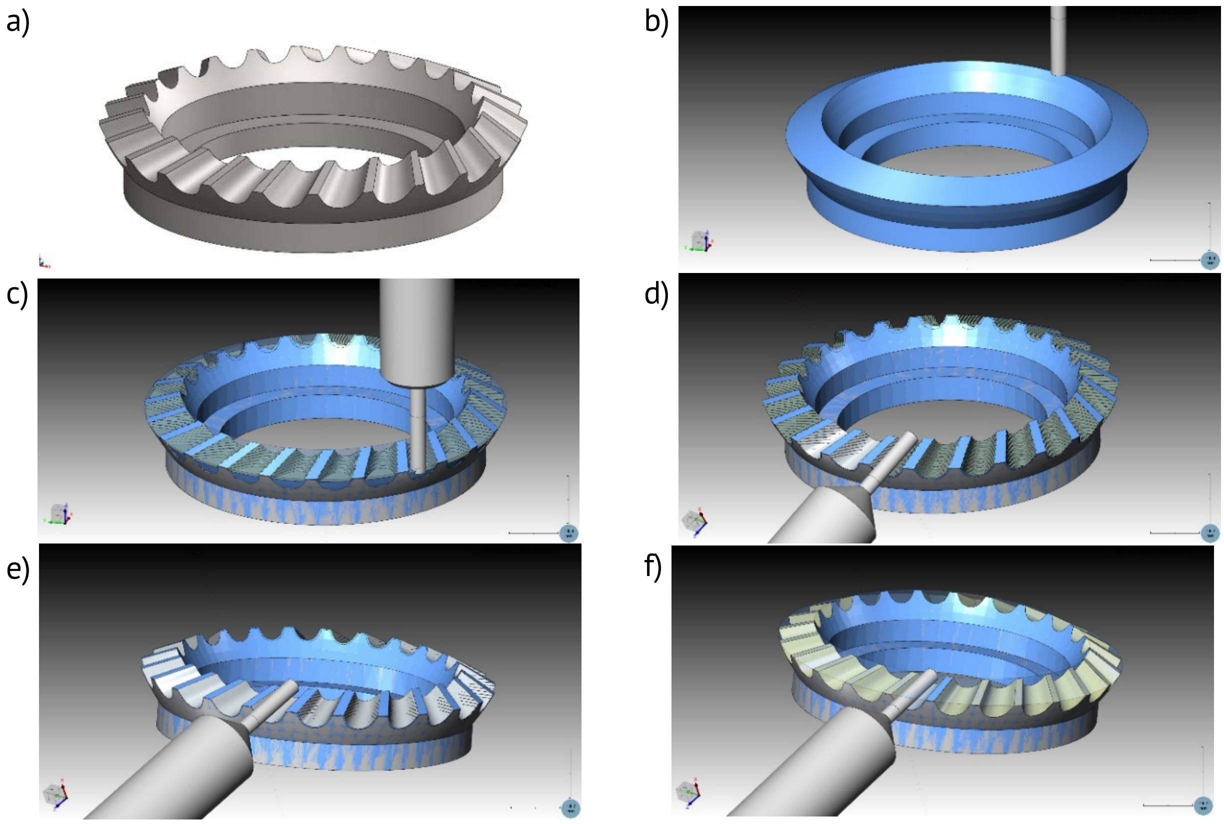

Figure 5. Manufacturing phases based on CAM modelling of the central toothed wheel with inclined teeth: CAD model (a); positioning of the blank (b); roughing of the toothed crown in 3 axes (c); 5-axis machining of the flank surface, preventive (d), intermediate (e) and final (f).

The third operation is the finishing operation - using the same machining method with constant steps - this time the machining is performed in constant steps with a spherical cutter with a radius for example of 3 mm and a pitch of 0.2 mm.

In Figure 5 (a) to (f) the manufacturing phases of the teeth of the central wheel with inclined teeth in the gearing $A^{D,\beta}_{CV-CV}$ by the process $G^{cil,\beta}_{m.ax}$ with cylindrical tool on numerically controlled multi-axial machine tools are shown: (a) CAD model; (b) - positioning of the blank; (c) - roughing of the toothed crown by three-axis milling; 5-axis machining of the flank surface, preventive (d), intermediate (e) and final (f).

3. Conclusions

1. The geometry of the concave-concave contact of the teeth in the gearing $A^{D,\beta}_{CV-CV}$ depends on the parametric configuration $[Z_g\text{-}\theta,\ \pm 1]$, and its bearing capacity - on the smallest possible difference in the radii of curvature of the conjugated profiles;

2. The degree of coverage of the teeth in the gearing $A^{D,\beta}_{CV-CV}$ results from the frontal coverage determined by the configuration parameters $[Z_g\text{-}\theta,\ \pm 1]$ and the longitudinal coverage depending on the angle of inclination β_g and nutation θ;

3. The relative frictional sliding between the flanks of the inclined teeth can be reduced by: rational choice of configuration parameters $[Z_g\text{-}\theta,\ \pm 1]$, changing the shape of the teeth by cutting (shortening) their height so as to increase the quota of pure rolling of the conjugated teeth due to spherospatial motion;

4. The generation of inclined teeth on numerically controlled multi-axial machine tools ensures the reduction of time and material resources for capitalizing new (non-standard) gears ensuring superior surface quality and high dimensional accuracy.

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