# On generalization of expressibility in 5-valued logic

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The function $f$ (the principal derivate operation) of algebra $\mathfrak{A}$ is called parametrically expressible via a system of functions $\Sigma$ of $\mathfrak{A}$, if there exists the functions $g_1, h_1, ..., g_m, h_m$ which are expressed explicitly via $\Sigma$ using superpositions, such that the predicate $f(x_1...x_n) = x_{n+1}$ is equivalent to the predicate $\exists t_1 ... \exists t_k ((g_1 = h_1) \wedge ... \wedge (g_m = h_m))$ on the algebra $\mathfrak{A}$.
Let us consider the pseudo-Boolean algebra $\langle M; \wedge, \vee, \supset, \neg \rangle$, where $\supset$ is relative pseudo-complement, and $\neg$ is pseudo-complement.
We say that the system of pseudo-Boolean terms on the set of variables $\mathcal{X}$ ($\Omega$ – words over $\mathcal{X}$) is parametrically complete in algebra $\langle M; \Omega \rangle$, if we can parametrically express the operations from $\Omega$ via functions expressed by terms over $\Sigma$. The function $f(x_1, ..., x_n)$ of $M$ preserves the predicate (relation) $R(x_1, ..., x_m)$ if for any possible values $x_{ij} \in M$ $(i = 1, ..., m; j = 1, ..., n)$ from the truth of $R(x_{11}, x_{21}, ..., x_{n1}), ..., R(x_{1n}, x_{2n}, ..., x_{mn})$ follows the truth of
$R(f(x_{11}, x_{12}, ..., x_{1n}), ..., f(x_{n1}, x_{n2}, ..., x_{nm}))$.
The centralizer $\langle f(x_1, ..., x_n) \rangle$ coincides with the set of all functions of $M$, which preserve the predicate $f(x_1, ..., x_n) = x_{n+1}$, where the variable $x_{n+1}$ differs from $x_1, ..., x_n$.
We examine the 5-valued pseudo-Boolean algebra $Z_5 = \langle \{0, \rho, \tau, \omega, 1\}; \wedge, \vee, \supset, \neg \rangle$, where $0 < \rho < \omega < 1$, $0 < \tau < \omega < 1$, $\rho$ and $\tau$ are incomparable elements. The algebra $Z_3 = \langle \{0, \omega, 1\}; \wedge, \vee, \supset, \neg \rangle$ is a subalgebra of $Z_5$.
Let us define the function $\varphi(p)$ on $Z_5$ as follows:

$$\varphi(0) = 0, \varphi(\rho) = \tau, \varphi(\tau) = \rho, \varphi(\omega) = \varphi(1) = 1.$$

The logic of the algebra $\mathfrak{A}$ is defined as the set of all formulas that are true on $\mathfrak{A}$, i.e. formulas identically equal to the greatest element 1 of this algebra.

**Theorem 1.** *A system of formulas $\Sigma$ is parametrically complete in the logic of algebra $Z_5$ iff $\Sigma$ is parametrically complete in the logic of subalgebra $Z_3$ and the system $\Sigma$ is not included into the centralizer $\langle \varphi(p) \rangle$ on algebra $Z_5$.*