Subnormalizing Extensions and D-structures

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A ring S with identity 1 is a *subnormalizing extension* of a subring R with the same identity if for some finite subset $\{x_1, x_2, \ldots, x_n\}$ or some countable subset $\{x_i \mid i = 1, 2, \ldots\}$ of elements of S, we have $S = \sum x_i R = \sum R x_i$ and $\sum_{j=1}^{i} x_j R = \sum_{j=1}^{i} R x_j$ for each i. A D-*structure* on a ring A with identity 1 is a family of self-maps σ_{gh} indexed by pairs of elements of a monoid G with identity e satisfying a large collection of rather natural conditions which allow the construction of a generalized monoid ring $A\langle G; \sigma\rangle$ with multiplication given by $(a \cdot x)(b \cdot y) = a \sum_{z \in G} \sigma_{x,z}(b) \cdot zy$.
In this talk all rings have identity but need not be commutative.
In many cases the ring $A\langle G; \sigma\rangle$ is a subnormalizing extension of A generated by G. On the other hand, if S is a subnormalizing extension of R, the x_i form a multiplicative submonoid of S, and generate S as a free left R-module, then as for each i and each $r \in R$ we have $x_i r = \sum_{j=1}^{i} r_{ij} x_j$ for unique elements r_{ij} of R, we can define $\sigma_{ij}(r) = r_{ij}$ when $i \geq j$ and 0 when $i < j$ and thereby obtain a collection of self-maps of R which turn out to satisfy most of the requirements of a D-structure. For several types of monoid they do form a D-structure. Whether this is so in all cases remains an open question.

Bibliography

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