Hausdorff extensions

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Any space is considered to be a Hausdorff space. We use the terminology and notations from [3, 1, 2].

Let τ be an infinite cardinal.

A point $x \in X$ is called a $P(\tau)$-point of the space X if for any non-empty family γ of open subsets of X for which $x \in \cap\gamma$ and $|\gamma| < \tau$ there exists an open subset U of X such that $x \in U \subset \cap\gamma$. If any point of X is a $P(\tau)$-point, then we say that $P(\tau)$-space.

Fix a set Φ of almost disjoint τ-centered families of subsets of the set E. We put $e_\Phi E = E \cup \Phi$. On $e_\Phi E$ we construct two topologies.

Topology $T^s(\Phi)$. The basis of the topology $T^s(\Phi)$ is the family $\mathcal{B}^s(\Phi) = \{U_L = L \cup \{\eta \in \Phi : H \subset L \textit{ for some } H \in \eta\} : L \subset E\}$.

Topology $T_m(\Phi)$. For each $x \in E$ we put $B_m(x) = \{\{x\}\}$. For every $\eta \in \Phi$ we put $B_m(\eta) = \{V_{(\eta,L)} = \{\eta\} \cup L : L \in \eta\}$. The basis of the topology $T_m(\Phi)$ is the family $\mathcal{B}_m(\Phi) = \cup\{B_m(x) : x \in e_\Phi E\}$.

Theorem 1. *The spaces* $(e_\Phi E, T^s(\Phi))$ *and* $(e_\Phi E, T_m(\Phi))$ *are Hausdorff zero-dimensional extensions of the discrete space* E*, and* $T^s(\Phi) \subset T_m(\Phi))$*. In particular,* $(e_\Phi E, T^s(\Phi)) \leq (e_\Phi E, T_m(\Phi))$*.*

Theorem 2. *The spaces* $(e_\Phi E, T^s(\Phi))$ *and* $(e_\Phi E, T_m(\Phi))$ *are* $P(\tau)$*-spaces.*

Corollary 3. *If* $T^s(\Phi) \subset \mathcal{T} \subset T_m(\Phi))$*, then* $(e_\Phi E, \mathcal{T})$ *is a Hausdorff extension of the discrete space* E*, and* $(e_\Phi E, T^s(\Phi)) \leq (e_\Phi E, \mathcal{T}) \leq (e_\Phi E, T_m(\Phi))$*.*

Theorem 4. *The space* $(e_\Omega E, T^s(\Omega))$*, where* Ω *is the set of well-ordered almost disjoint* τ*-centered families, is a zero-dimensional paracompact space with character* $\chi(e_\Omega E, T^s(\Omega)) = \tau$ *and weight* $\Sigma\{|E|^m : m < \tau\}$*.*

Consider the Hausdorff extension rE of the space E. We put $e_{rE}X = X \cup (rE \setminus E)$. In $e_{rE}X$ we construct the topology $\mathcal{T} = T(\gamma, E, \xi_\mu, \tau)$.

Theorem 5. *The space* $(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau))$ *is a Hausdorff extension of the space* X*.*

Theorem 6. *If* rE *is a* $P(\tau)$*-space, then* $(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau))$ *is a* $P(\tau)$*-space too. Moreover,* $\chi(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau)) = \chi(X) + \chi(rE)$ *and* $w(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau)) = w(X) + w(rE)$*.*

Theorem 7. *Assume that the spaces* rE *and* X *are zero-dimensional, and the sets* $H_{(\mu,\alpha)}$ *are open-and-closed in* X*. Then: 1.* $(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau))$ *is a zero-dimensional space.*

2. The space $(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau))$ *is paracompact if and only if the spaces* rE *and* X *are paracompact.*

Bibliography

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